

A UTILITY PROGRAM FOR THE ANALYSIS
OF RISK (UPFAR)

By

MOHAMED SADEK EID
4

Bachelor of Science in Engineering
Cairo University
Cairo, Egypt
1960

Master of Engineering
Dartmouth College
Hanover, New Hampshire
1968

Industrial Engineer
Columbia University
New York, New York
1973

Submitted to the Faculty of the Graduate College
of the Oklahoma State University
in partial fulfillment of the requirements
for the Degree of
DOCTOR OF PHILOSOPHY
July, 1974

Thesis
1974①
E34u
cop.2

MAR 13 1975 MAR 13 1975 MA

A UTILITY PROGRAM FOR THE ANALYSIS OF RISK (UPFAR) RISK (UPFAR) RISK (UPFAR)

Thesis Approved: Thesis Approved: Thesis Approved:

Thesis Advisor: James K. Egan
Joe H. Magin
Donald R. Magin

Dean of the Graduate College: M. Palm
N. D. Smith

902067 902067 902067

ACKNOWLEDGEMENTS

I would like to express my appreciation to the many individuals whose help, encouragement, and guidance have made it possible for me to attain this level of education. I would like to express my appreciation to my committee: Dr. Hamed K. Eldin, Dr. Donald W. Grace, Dr. Joe H. Mize, and Dr. M. Palmer Terrell. Their teaching, support and encouragement together with their challenges have stimulated my interests in my work. I would like to express my gratitude to Dr. Hamed K. Eldin, not only for serving as my committee chairman and thesis adviser, but also for his invaluable support and guidance during the last seven years. I owe a great deal of my success in business and education to his invaluable advice.

I would like to thank Mr. Lee W. Falls, NASA, Marshal Space Flight Center, for his support during the development of this research. I would like to also thank NASA for supplying and permitting me to use Mr. Falls' program: "A Computer Program for Standard Statistical Distributions," which has been very helpful in my research.

My wife, Susan, and daughter, Soha, have offered me their understanding, sacrifices, support, and love that are essential for any success and progress. To them I am deeply grateful.

Mrs. Margaret Estes has done an excellent job in typing this thesis. I am grateful for her efforts.

TABLE OF CONTENTS

Chapter	Page
I. INTRODUCTION	1
1.1 Research Objectives	1
1.2 Scope of the Investigation	3
II. RISK ANALYSIS RESEARCH DESCRIPTION AND LITERATURE REVIEW	6
2.1 Classification of Risk Analysis Research	6
2.2 Previous Research	8
III. RISK ANALYSES CONCEPTS	10
3.1 Risk Measurement	10
3.2 Investment Selection	11
3.3 Utility Models	14
3.4 Probability Distributions	17
IV. SYSTEM CONSTRUCTION AND VALIDATION	31
4.1 Preliminary Considerations	31
4.2 UPFAR: <u>Utility Program For the Analysis</u> <u>of Risk--Basic System Structure</u>	32
4.3 System Validation	41
4.4 Notes on UPFAR	44
V. SAMPLE CASE STUDY--ANALYSIS OF INVESTMENT IN OUTLET DEPOTS	46
5.1 The INCO Company Investment Problem	46
5.2 Analysis of the Investment Opportunity-- Data Collection	46
5.3 Preparation of Input for INMO	60
5.4 Preparing the Information for an UPFAR Run	60
5.5 Analysis of the Simulation Results	62
5.6 Distribution of the Net Present Value of the Investment at 10% Rate	63
5.7 Utility of the Investment	65
VI. CONCLUSIONS AND RECOMMENDATIONS	67
REFERENCES	71
A SELECTED BIBLIOGRAPHY	74

Chapter	Page
APPENDIX A--UPFAR USER'S GUIDE	79
A.1 PRODIP User's Guide	81
A.2 GUSS User's Guide	92
A.3 INMO User's Guide	106
A.4 RUM User's Guide	116
APPENDIX B--INPUT DATA SHEETS AND UPFAR MODULES' INPUT/OUTPUT . .	121
B.1 PRODIP Input Data Sheets	122
B.2 PRODIP Sample Run Input	125
B.3 PRODIP Sample Run Output	128
B.4 GUSS Input Data Sheets	147
B.5 GUSS Sample Model Subroutine	153
B.6 GUSS Model Subroutine Input	155
B.7 GUSS Sample Run Output	164
B.8 INMO Input Data Sheets	172
B.9 RUM Input Data Sheet	180
B.10 The INCO Company Investment Problem-- GUSS Input	182
B.11 The INCO Company Investment Problem-- GUSS Output	200
B.12 The INCO Company Investment Problem-- PRODIP Input	214
B.13 The INCO Company Investment Problem-- PRODIP Output	217
B.14 The INCO Company Investment Problem-- RUM Input	231
B.15 The INCO Company Investment Problem-- RUM Output	233

LIST OF TABLES

Table	Page
I. Lease Value for Land Necessary for the INCO Company Project	48
II. Working Capital Needed for the INCO Company Project . .	48
III. Investments in Buildings Needed for the INCO Company Project	49
IV. Investment in Heavy Equipment Needed for the INCO Company Project	49
V. Investment in Light Equipment Needed for the INCO Company Project	50
VI. Allowed Depreciation for Investment Items	50
VII. Start of Selling for Product PRODONE	52
VIII. Start of Selling for Product PRODTWO	52
IX. Maturity Sales Volume for Product PRODONE	53
X. Maturity Sales Volume for Product PRODTWO	53
XI. Growth Period to Maturity for Product PRODONE	54
XII. Growth Period to Maturity for Product PRODTWO	54
XIII. Selling Price Per Unit of PRODONE	55
XIV. Selling Price Per Unit of PRODTWO	55
XV. Direct Cost Per Unit of PRODONE	56
XVI. Direct Cost Per Unit of PRODTWO	56
XVII. Maturity Level of Fixed Expenses Per Year for the INCO Project	57
XVIII. Maturity Level of Advertising Expenses Per Year for the INCO Project	57
XIX. Starting Time for Fixed Expenses	58

Table	Page
XX. Starting Time for Advertising Expenses	58
XXI. Growth Period to Maturity for Fixed Expenses	59
XXII. Growth Period to Maturity for Advertising Expenses . . .	59
XXIII. Tax Rate	61

LIST OF FIGURES

Figure	Page
1. Summary: Applications of Continuous Statistical Distributions	29
2. Summary: Applications of Discrete Statistical Distributions	30
3. <u>Utility Program For Analysis of Risk</u> (UPFAR)-Flow Chart . .	33
4. <u>Probability Distribution Fitting Program</u> (PRODIP)-Flow Chart	35
5. <u>General Utility Simulation System</u> (GUSS)-Flow Chart	38
6. <u>Investment Model</u> (INMO)-Flow Chart	42
7. <u>Routine for Utility Measurement</u> (RUM)-Flow Chart	43
8. Probability Distribution of the NPV at 10% Discount Rate for the INCO Company Investment Project	64
9. <u>Utility Program For Analysis of Risk</u> (UPFAR)-Flow Chart . .	82
10. <u>Probability Distribution Fitting Program</u> (PRODIP)-Flow Chart	83
11. <u>General Utility Simulation System</u> (GUSS)-Flow Chart	93
12. Simple Investment Model	105
13. <u>Investment Model</u> (INMO)-Flow Chart	109
14. <u>Routine for Utility Measurement</u> (RUM)-Flow Chart	117

CHAPTER I

INTRODUCTION

Economic decisions regarding capital investments involve choosing between different proposed alternatives. Each of these alternatives includes events and outcomes that would occur in the future. In making such decisions, there is always an amount of risk involved regarding the future outcomes of the different alternatives. The amount of risk involved is an important consideration in the evaluation of the proposed investments. Neglecting the risk may result in faulty decisions. Furthermore, risk information permits management to weigh more precisely the possible consequences of the proposed investments.

1.1 Research Objectives

The major objective of this research is to develop a simulation model that can be used for the understanding of the risk inherent in investment decisions as well as quantifying that risk. The risk involved in an investment is then included in the selection criteria for new investment opportunities. It can also be considered in the ordering and ranking of competing investments.

In general, some analysts may present the results of their evaluation of an investment opportunity in the form of the net present value (NPV) of the investment for a given minimum acceptable rate of return (R/R). Other analysts present their results in the form of a rate of

return that the investment opportunity is expected to realize. Different decision makers would favor one of these two criteria, or another one, as a measure of the acceptability of the investment opportunity. Based on the author's preference, this research utilizes the net present value as the measurement criterion. Arguments in favor of one criterion against another are not the subject of this research. The model, however, can be extended to provide the rate of return and/or other criteria if so desired.

Estimates of the economic parameters of an investment are not the only factors that affect the acceptability of the investment. Other factors, quantifiable and non-quantifiable can be more critical to the extent that economic justifications become unimportant.

Uncertainties about future outcomes of some of the economic parameters of an investment can further complicate the decision making process. The net present value and the rate of return calculations are based on complex compounding formulae that give the resultant of the interaction of the different investment components. This complexity makes it difficult to include uncertainty assumptions about the investment components in the final estimate of the NPV or R/R in a neat closed mathematical formula. Research literature was published giving different formulae or rules for investments under uncertainty for special cases and/or simplified assumptions (see Chapter II). However, solutions to special cases cannot be generalized for many of the practical applications. Simplifying assumptions can result, in most cases, in changing the characteristics of the original problem making the results of the simplified model in many cases misleading.

Simulation is an effective technique for generating the convolutions of different probability distributions. This research will utilize simulation in generating the parameters of the resulting distribution for the NPV of an investment. The NPV distribution is the convolution of the investment's components' distributions.

The resultant NPV distribution can be used as the basis for comparison with other competing investments. The parameters of the NPV distribution can also be substituted in the utility model of the investor to obtain the utility measure of the investment. Different utility models are available in literature and can be used for the purpose of investment evaluation and selection. The model presented in this research will include some of these utility models for evaluating an investment opportunity if so desired.

1.2 Scope of the Investigation

This research is directed toward developing an integrated simulation model that would:

- Determine probability distributions of investment components' parameters based on empirical data.
- Simulate the investment's future cash flows to give the distribution of the NPV (which is the convolution of the distributions of the different components and their mathematical relationships) as well as its parameters.
- Evaluate the investment's utility based on different utility models to provide a common risk measurement among competing investments.

The system presented here, UPFAR (Utility Program For Analysis of Risk), is therefore divided into four major parts:

1. Probability Distribution Fitting Program (PRODIP),
2. General Utility Simulation System (GUSS),
3. Investment Problem Model Routine (INMO), and
4. Routine for Utility Measurement (RUM).

1.2.1 Probability Distribution Fitting Program (PRODIP)

Determining the probability distribution of an investment parameter is an important phase of risk analysis. In some cases, empirical data of previous experiences with similar parameters do not exist. In such cases, subjective probabilities or distributions may be used. In other cases, historical data exist. In these cases, these data should be used to determine the best distribution that fits these data. The PRODIP module presented in this research generates the distribution from the Pearson family that best fits the data. Furthermore, if it is felt that a particular probability distribution (normal, Weibull, etc.) should be used to represent the parameter, the algorithm, when instructed, would give the best fit to the data of that distribution.

1.2.2 General Utility Simulation System (GUSS)

GUSS is a general purpose Monte Carlo simulation system. GUSS is responsible for sampling from the parameters' distributions, linking the data to the Investment Problem Model Routine (INMO), performing all the necessary housekeeping of data and results, and presenting the final results with their distributions and parameters. It also, if requested,

would link with the Utility Evaluation Algorithm (RUM) to present a final pricing of an investment opportunity.

1.2.3 Investment Problem Model Routine (INMO)

INMO is the routine that gives the performance relationships and model of the investment problem. The routine presented in this research can be used to solve a wide class of investment problems. However, special problems may require special routines that can be developed. Linkage of an investment model routine with the Simulation Model is a simple task, which is a characteristic feature of GUSS.

1.2.4 Routine for Utility Measurement (RUM)

RUM is the routine that includes different utility functions that may be used to calculate a utility value for the investment under study. Three utility models are included in RUM. The distribution parameters of the net present value of an investment are entered into RUM. RUM generates the different utility indexes. These indexes are used in the decision making process for determining the acceptability of the investment.

CHAPTER II

RISK ANALYSIS RESEARCH DESCRIPTION AND LITERATURE REVIEW

2.1 Classification of Risk Analysis Research

Risk in projects was recognized by conventional management in their decision making process. Lacking the models to account for the risk in the analysis, investment problems were initially solved as deterministic to estimate a most likely outcome and then augment the analysis by:

- Using a shorter payout period for recovering the initial investment.
- Using a higher rate of return than is normally accepted.
- Using conservative estimates of the uncertain variables.

Such heuristic approaches, however, may be critiqued as being misleading to the investment decision making process. For example, a highly optimistic estimate may be disastrous while a very conservative one can result in missing a profitable investment opportunity.

Mathematical models and simulation techniques can be used to evaluate a measure for the risk in an investment. When considering investment problems, two types of models are to be differentiated. These are the investment analysis models and the investment selection models. Investment analysis models do not include, within their formulation, criteria for comparisons between the different investments among themselves or against a standard. They rather evaluate the risk in an

investment. Investment selection models, on the other hand, take the risk measurements of the different projects, process them in the selection criteria for selecting these projects to be included in the investment portfolio. The research presented in this paper concerns itself to the investment analysis area.

Risk analysis models are investment analysis models that deal with cases where some of the cash flows of the investment are stochastic rather than deterministic. For risk analysis problems with independent cash flows, estimating the mean and variance of the NPV of the investment may be a simple task. However, when the cash flows of the investment are functionally and/or time dependent, the problem becomes more complex and would not lend itself to a mathematical model.

Mathematical models have been developed which utilize higher mathematics and theory of probability. Although theoretically sound, these models have not been widely implemented. This can be attributed to the assumptions and approximations they included to make the problem mathematically manageable. Such assumptions and approximations result in final solutions that are not representative of the original investment problem.

Simulation models have been tried by researchers and different simulation models were developed to solve the complex risk analysis problems. However, available models lack the integration of evaluating the probability distributions of the investment components, the simulation model and common measures of risk that can be used for investment selection. This is the major thrust of this research.

2.2 Previous Research

The Risk Analysis Library is rich with research. This section will not give an exhaustive listing of this library but will rather list important models.

English [1] and Solomon [2] proposed a variable discounting rate of return function to compensate for risk in investments. Hillier [3, 4, 5] developed an analytical model for risk analysis using probability theory. Hillier's model, however, assumes that the cash flows are both functionally and time independent to generate the expected value and variance of the investment's NPV. Hillier [3] also proposed an extension of the model for the case when some of the cash flows are time dependent but functionally independent. Canada and Wadsworth [6] developed a semi-graphical method as a solution to risk analysis problems where some of the cash flows are partially correlated functionally within a period, but perfectly correlated as to like cash flows among periods. Continuous compounding is used in this model to get a differentiable NPV function. The assumptions of time-invariant cash flow streams and the restriction to a one time investment at the beginning of the project life limit the application of the model. Horowitz [7] developed a model for the same case but for different cash flows. A major conclusion of the Horowitz's model is that it showed that the resultant NPV of the project was not normally distributed. Other studies either assumed normality directly or that normality was approached by the addition of the cash flow stream. Hess and Quigley [8] used Monte Carlo simulation to evaluate investments with independent cash flow. Hertz [9, 10] gives basic considerations for applying risk analysis in project selection. Bussey [11] used GPSS to simulate a special problem where the periodic cash flows are not

perfectly correlated, but are time-related through an exponential growth function. Bussey [12] also developed a quartic utility function including the first four moments of the NPV distribution as a general function describing both risk-averse investors and those who are not risk-avoiders. Reisman and Roe [13] developed formulae to evaluate investments for some special stochastic cases.

CHAPTER III

RISK ANALYSES CONCEPTS

3.1 Risk Measurement

When the parameters of an investment are all deterministic, one can use any of different formulae, developed in engineering economy references, to evaluate its worthiness. Similar formulae were developed by Reisman and Rao [13] for some special stochastic variations of the problem. To evaluate the risk in an investment, the following general model can be used:

$$NPV = \sum_{t=1}^n Y_t (1+r)^{-t} \quad (3.1)$$

where:

NPV = Net Present Worth of investments,

n = life of project,

r = company's minimum acceptable rate of return,

Y_t = vector of cash flows at period t,

= $[y_k]_t$, $k = 1, 2, \dots, m$, and

m = number of different types of cash flows.

As an example of cash flow types for model (3.1), y_1 can be investment in land, y_2 working capital, y_5 investment in a certain type of equipment, y_{10} revenue from certain sales, etc.

A cash flow, y_k , does not necessarily have to be a simple random variable, but can be the resultant of the interaction of different components, compounded together with simple or complex relationships. These components can assume deterministic or stochastic variables, dependent or independent. The life of the project and the rate of return of the company may also be assumed as random variables. As a result of these possible stochastic parameters included in the NPV calculation, model 3.1, the resulting NPV is also a random variable. The distribution of the NPV is dependent on the distribution of its component elements. Assumptions have been made by different researchers that the NPV distribution is normal. Such assumptions can sometimes be correct. However, the normality assumption is not generally true as in Horowitz's model [7].

3.2 Investment Selection

After determining the distribution of the NPV of the different investment opportunities, two major decisions may be involved:

- 1) To accept or reject the project.
- 2) If accepted, to rank those projects that are acceptable.

In the absence of a utility function, against which all projects can be evaluated, the acceptance or rejection of the project and the ranking of projects by the decision maker will be subjective in nature. However, consistent with a rational decision policy, the acceptance or rejection of a project "A", and the preference of a project "B" over "A" may be decided according to one or more of the non-exhaustive set of rules outlined in the following paragraph.

Denoting:

$f(x)$ = probability density function of x ,

$m_i(x) = i^{\text{th}}$ moment of x ,

$E(x) =$ expected value of x ,

$\sigma(x) =$ standard deviation of x ,

$\bar{R}_x \equiv$ maximum acceptable ratio on x ,

$\underline{R}_x \equiv$ minimum acceptable ratio on x ,

$V \equiv$ Net Present Value, NPV,

$L \equiv$ Loss,

$G \equiv$ Gain,

$\bar{L} \equiv$ maximum acceptable loss

$\underline{G} \equiv$ minimum acceptable gain,

$D \equiv$ odds of gain,

$\underline{D} \equiv$ minimum acceptable odds in favor of investment,

$U \equiv$ Utility,

$\underline{U} \equiv$ minimum acceptable utility.

3.2.1 Rejection of an Investment

Investment 'A' may be rejected if one or more of the following conditions occur:

a) Minimum acceptable NPV

$$E(V_A) < \underline{V} \quad (3.2)$$

where

$$E(V) = \int_{-\infty}^{\infty} V f(V) dV . \quad (3.3)$$

b) Maximum tolerable variance ratio

$$\frac{\sigma(V_A)}{E(V_A)} > \bar{R}_V \quad (3.4)$$

where

$$\sigma^2(V) = \int_{-\infty}^{\infty} V^2 f(V) dV . \quad (3.5)$$

c) Maximum allowable expected loss

$$E(L_A) > \bar{L} \quad (3.6)$$

where

$$E(L) = \int_{-\infty}^{\infty} |V| f(V) dV . \quad (3.7)$$

d) Maximum allowable loss ratio

$$\frac{E(L_A)}{E(V_A)} > \bar{R}_L . \quad (3.8)$$

e) Minimum acceptable gain

$$E(G_A) < \underline{G} \quad (3.9)$$

where

$$E(G) = \int_0^{\infty} V f(V) dV . \quad (3.10)$$

f) Minimum allowable gain ratio

$$\frac{E(G_A)}{E(V_A)} < \underline{R}_G . \quad (3.11)$$

g) Minimum odds on gain

$$D_A < \underline{O} \quad (3.12)$$

where

$$D = \frac{p(G)}{p(L)} \quad (3.13)$$

and

$$p(L) = \int_{-\infty}^0 f(V) dV \quad (3.14)$$

$$p(G) = \int_0^{\infty} f(V) dV . \quad (3.15)$$

h) Minimum acceptable utility

$$U_A < \underline{U} . \quad (3.16)$$

3.2.2 Ranking of Investments

Investment B is preferable than A if one or more of the following conditions occur:

$$a) \quad E(V_A) < E(V_B) \quad (3.17)$$

$$b) \quad \frac{\sigma(V_A)}{E(V_A)} > \frac{\sigma(V_B)}{E(V_B)} \quad (3.18)$$

$$c) \quad E(L_A) > E(L_B) \quad (3.19)$$

$$d) \quad \frac{E(L_A)}{E(V_A)} > \frac{E(L_B)}{E(V_B)} \quad (3.20)$$

$$e) \quad E(G_A) < E(G_B) \quad (3.21)$$

$$f) \quad \frac{E(G_A)}{E(V_A)} < \frac{E(G_B)}{E(V_B)} \quad (3.22)$$

$$g) \quad D_A < D_B \quad (3.23)$$

$$h) \quad U_A < U_B \quad (3.24)$$

3.3 Utility Models

When a utility function that represents the investor's attitude is available, different investment opportunities are evaluated against this function. An investment opportunity is accepted if it scores higher than a minimum utility measure decided by the investor. Also, a project "A" is preferred to project "B" if the utility of "A" is higher than that of "B."

Different utility theorems and models have been developed by researchers. To discuss these models, let us define the following nomenclature:

$E(U)$ = expected utility of the return of a particular portfolio,

μ = mean return for the portfolio,

m_i = i th moment,

σ = standard deviation of the return for the portfolio,

a, b, c, d = constants, and

$U(V)$ = utility of V .

3.3.1 Hicks and Allen

Hicks and Allen [14] constructed a theory of consumer behavior without assuming that utility was a measurable quantity. Their ordinal utility theorem is based on the assumption that a consumer has a scale of preferences when ranking different collections of goods.

3.3.2 Marshal

The cardinal utility by Marshal [15] recognizes that utility is a psychic quantity which is measurable and quantifiable.

3.3.3 Von Neumann and Morgenstern

The utility theorem of Von Neumann and Morgenstern [16] is based on a series of axioms of rational behavior for assigning utility to monetary payoffs with varying degrees of risk. According to the theorem, a series of gambles is presented to the decision maker and his responses are then plotted to define his utility function.

3.3.4 Markowitz

In his Security Portfolio Selection model, Markowitz [17] used a utility function of the form:

$$E(U) = \mu - a \cdot \sigma^2 \quad (3.25)$$

where a here is the coefficient of risk aversion.

3.3.5 Farrer and Freund

Farrer [18] and Freund [19] used a utility function of the form:

$$U(V) = 1 - e^{-aV} \quad (3.26)$$

3.3.6 Cramer and Smith

Cramer and Smith [20] used a model of the form:

$$E[U(V)] = \mu - a\sigma^b I^c \quad (3.27)$$

where I is the amount of investment in the project and

$$\mu = E(V).$$

3.3.7 Bussey

Bussey [11] developed a quartic utility function as a general model for both risk-averse investors as well as non-risk avoiding ones. His model includes third and fourth moments as follows:

$$U(V) = aV - bV^2 + cV^3 - dV^4 \quad (3.28)$$

$$\therefore E[U(V)] = aE(V) - bE(V^2) + cE(V^3) - dE(V^4) \quad (3.29)$$

where A, B, C, D are constants. Substituting for:

$$E(V) = \mu \quad (3.30)$$

$$E(V^2) = \sigma^2 + \mu^2 \quad (3.31)$$

$$E(V^3) = 3\sigma^2\mu + \mu^3 + m_3 \quad (3.32)$$

$$E(V^4) = \mu^4 + 6\sigma^2\mu^2 + 4m_3\mu + m_4 \quad (3.33)$$

$$\begin{aligned} \therefore E[U(V)] &= -d\mu^4 + c\mu^3 - (b+6d\sigma^2)\mu^2 \\ &\quad + (a+3c\sigma^2-4dm_3)\mu \\ &\quad - (b\sigma^2 - cm_3 + dm_4) \end{aligned} \quad (3.34)$$

where

$$\mu = E(V) \quad (3.35)$$

$$\sigma^2 = E(V-\mu)^2 \quad (3.36)$$

$$m_3 = E(V^3) \quad (3.37)$$

$$m_4 = E(V^4) \quad (3.38)$$

Utility functions have been critiqued for different reasons.

Swalm [21] questions the stability of utility functions over time.

Raiffa [22, 23] and others indicated that utility models are normative, indicating how the decision-maker's behavior should be rather than how his actual behavior is. It should be commented that utility theory applications have not matured and further research is still needed in this area.

3.4 Probability Distributions

A major task in risk analysis is the assignment of probabilities and distributions to the different parameters of the investment problem. Appropriate considerations should be given to the physical processes that govern the properties of a given distribution function before assigning it to a stochastic variable, e.g., continuous, discrete, etc. Three approaches could be used for assigning probabilities depending on the problem:

- a) In some cases, the shape of the probability distribution of the stochastic variable is assumed based on previous experiences or speculations about the variable. The statistical parameters of the assumed distribution are also assumed in the same manner.
- b) In this case, subjective probabilities may be estimated or assumed based on previous experiences or speculations about the variable. These subjective probabilities can be used discretely, interpolated for simulating the continuous case, or can be used as the basis for a maximum entropy distribution assumption.

- c) In cases where empirical or historical data are available, these data can be used to determine the frequency or probability distribution of the stochastic variable under study.

Three methods are available to generate a distribution for a stochastic variable based on the available information.

3.4.1 Maximum Entropy Distributions [24]

Based on Jaynes' principle of maximum entropy, the minimally prejudiced probability distribution is that which maximizes the function:

$$\Phi = - \sum_i p_i \ln p_i . \quad (3.39)$$

where Φ is the entropy and p_i is the probability of outcome i . Now let us denote: p_k = probability of x_k and $E(x)$ = expected value of x . The maximum entropy formalism gives the different distributions that fit the available information.

- a) Using only the information that

$$\sum_{k=0}^m p_k = 1 \quad (3.40)$$

the uniform distribution would be the minimally prejudiced distribution for this information.

- b) Given the information that

$$\sum_{k=0}^{\infty} p_k = 1 \quad (3.41)$$

and

$$\sum p_k x_k = E(x) \quad (3.42)$$

the Exponential distribution is then the minimally prejudiced distribution for this information.

- c) Given the information that

$$\sum_{k=0}^{\infty} p_k = 1, \quad (3.43)$$

$$\sum_{k=0}^{\infty} p_k x_k, \quad (3.44)$$

and

$$\sum_{k=0}^{\infty} p_k x_k^2 = E(x^2). \quad (3.45)$$

The Truncated Gaussian distribution is the minimally prejudiced for this information. In the case when the range of x is $-\infty \leq x \leq \infty$, the distribution in this case is the Normal Gaussian.

- d) Given the information that

$$\sum_{k=0}^{\infty} p_k = 1 \quad (3.46)$$

$$\sum_{k=0}^{\infty} p_k x_k = E(x) \quad (3.47)$$

$$\sum_{k=0}^{\infty} p_k \ln x_k = E(\ln x). \quad (3.48)$$

The Gamma distribution is the minimally prejudiced distribution for this information.

- e) Given the information that

$$0 \leq x_k \leq 1, \quad (3.49)$$

$$\sum_{k=0}^1 p_k = 1, \quad (3.50)$$

$$\sum_{k=0}^1 p_k \ln x_k = E(\ln x), \quad (3.51)$$

$$\sum_{k=0}^1 p_k \ln(1-x_k) = E(\ln(1-x)) . \quad (3.52)$$

The Beta distribution is the minimally prejudiced distribution for this information.

f) Given the information that

$$\sum_{i=1}^{\infty} p_k = 1 \quad (3.53)$$

$$\sum_{i=1}^{\infty} p_k \ln x_k = E(\ln x_k) \quad (3.54)$$

$$\sum_{i=1}^{\infty} p_k x_k^B = E(x_k^B) . \quad (3.55)$$

This leads to a family of distributions of which the Weibull distribution is a special case.

3.4.2 Fitting an Assumed Distribution

Having empirical data about a given parameter, one can try to fit these data to one or more of the known statistical distributions. The acceptance or rejection of a specific distribution to the data can be based on statistical tests for "goodness of fit." Let us denote:

$g(x)$ = probability density function for continuous distribution,

$p(x)$ = discrete density function,

$H(x)$ = cumulative distribution function, and

$m_i = i^{\text{th}}$ moment of the distribution.

Following is a list of possible statistical distributions and the estimation of their parameters from sample data.

a) Uniform Distribution:

$$g(x) = \frac{1}{b-a} \quad (3.56)$$

$$a < x < b \quad (3.57)$$

b) Exponential Distribution:

$$g(x) = \lambda_1 \exp(-\lambda_1 x) \quad (3.58)$$

$$x > 0 \quad (3.59)$$

$$\lambda_1 = \frac{1}{m_1} > 0 \quad (3.60)$$

c) Normal Distribution:

$$g(x) = \frac{1}{\sqrt{2\pi} \sigma_x} \exp\left(-\frac{(x-\mu_x)^2}{2\sigma_x^2}\right) \quad (3.61)$$

$$-\infty \leq x \leq \infty \quad (3.62)$$

$$-\infty < \mu_x < \infty \quad (3.63)$$

$$\sigma_x > 0 \quad (3.64)$$

d) Lognormal Distribution:

$$g(y) = \frac{1}{\sqrt{2\pi} \sigma_y} \exp\left[-\frac{1}{2}\left(\frac{y-\mu_y}{\sigma_y}\right)^2\right] \quad (3.65)$$

$$y = \ln x \quad (3.66)$$

$$x > 0 \quad (3.67)$$

$$-\infty < y < \infty \quad (3.68)$$

$$-\infty < \mu_y = m_1 < \infty \quad (3.69)$$

$$\sigma_y > 0 \quad (3.70)$$

e) Fisher-Tippet I:

$$g(x) = \alpha \exp[-y - \exp(-y)] \quad (3.71)$$

$$y = \alpha(x - \mu_x) \quad (3.72)$$

$$-\infty < x < \infty \quad (3.73)$$

$$\alpha = \sqrt{6\pi}/S_x > 0 \quad (3.74)$$

$$-\infty < \mu_x = m_1 - 0.577216649/\alpha < \infty \quad (3.75)$$

f) Fisher-Tippett II:

$$g(x) = \frac{\gamma}{\beta(x/\beta)^{\gamma-1}} \exp\left[-(x/\beta)^{-\gamma}\right] \quad (3.76)$$

$$x \geq 0 \quad (3.77)$$

$$\gamma > 0; \text{ where } \frac{N}{\gamma} - \sum_{i=1}^k (\ln x_i) f_i + \frac{\sum_{i=1}^k (x_i^{-\gamma} \ln x_i) f_i}{\sum_{i=1}^k x_i^{-\gamma} f_i} \quad (3.78)$$

$$\beta > 0; \text{ where } \frac{N}{\beta} - \beta^{\gamma-1} \sum_{i=1}^k x_i^{-\gamma} f_i \quad (3.79)$$

g) Weibull Distribution:

$$g(x) = \left(\frac{\gamma}{\theta}\right) x^{\gamma-1} \exp\left[-\frac{x^{\gamma}}{\theta}\right] \quad (3.80)$$

$$x \geq 0 \quad (3.81)$$

$$\gamma > 0; \text{ where } \frac{\sum_{i=1}^k x_i^{\gamma} (\ln x_i) f_i}{\sum_{i=1}^k x_i^{\gamma} f_i} - \frac{1}{\gamma} - \frac{1}{N} \left[\sum_{i=1}^k (\ln x_i) f_i \right] = 0 \quad (3.82)$$

$$\theta > 0; \text{ where } \theta = \frac{\sum_{i=1}^k x_i^{\gamma} f_i}{N} \quad (3.83)$$

h) Beta Distribution:

$$g(x) = \frac{\Gamma(\lambda + \theta)}{\Gamma(\lambda) \Gamma(\theta)} x^{\lambda-1} (1-x)^{\theta-1} \quad (3.84)$$

$$0 \leq x \leq 1 \quad (3.85)$$

$$\theta = \frac{(1 - \mu_x)}{\sigma_x^2} [\bar{x}(1 - \bar{x}) - \sigma_x^2] > 0 \quad (3.86)$$

$$\lambda = \frac{\bar{x}\theta}{1-\bar{x}} > 0 \quad (3.87)$$

i) Poisson Distribution:

$$p(x) = \frac{e^{-m} m^x}{x!} \quad (3.88)$$

$$x = 0, 1, 2, \dots \quad (3.89)$$

$$m = m_1 > 0 \quad (3.90)$$

j) Truncated Poisson Distribution:

$$p(x) = \frac{e^{-m} m^x}{x!} \left(\frac{1}{1 - \exp(-m)} \right) \quad (3.91)$$

$$x = 1, 2, 3, \dots \quad (3.92)$$

$$m > 0 \quad (3.93)$$

where

$$m / (1 - \exp(-m)) = m_1 \quad (3.94)$$

k) Binomial Distribution:

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x} \quad (3.95)$$

$$x = 0, 1, 2, \dots, n \quad (3.96)$$

$$n = 1, 2, 3, \dots \quad (3.97)$$

$$0 \leq p \leq 1 \quad (3.98)$$

l) Negative Binomial Distribution:

$$p(x) = \frac{\Gamma(x+k) \Gamma(k)}{x!} p^k (1-p)^x \quad (3.99)$$

$$x = 0, 1, 2, \dots \quad (3.100)$$

$$k = \frac{m_1^2}{m_2 - m_1} > 0 \quad (3.101)$$

$$0 \leq p = k / (k + m_1) \leq 1 \quad (3.102)$$

m) Truncated Negative Binomial Distribution:

$$p(x) = \frac{\Gamma(x+k)}{x!} \cdot \Gamma(k) \left[p^k (1-p)^x / (1-p^k) \right] \quad (3.103)$$

$$x = 1, 2, 3, \dots \quad (3.104)$$

$$0 \leq p = \left(\frac{m_1}{m_2} \right) \left(1 - \frac{N_1}{N} \right) \leq 1 \quad (3.105)$$

$$N_1 = \text{frequency of } (X = 1) \quad (3.106)$$

$$k = (pm_1 - N_1/N)/(1 - p) > 0 \quad (3.107)$$

3.4.3 Pearson System [24, 25, 26]

Pearson showed that a group of distribution families can be generated as a solution to the differential equation:

$$\frac{d g(x)}{dx} = \frac{(x-a) g(x)}{b_0 + b_1 x + b_2 x^2 + \dots} \quad (3.108)$$

Each family of these distributions can be generated by the proper choice of a and b_i 's, the Pearson Parameters, which are a function of the sample moments. The solution of Equation (3.108) leads to a large number of distribution families, including the normal, Beta (Pearson Type I), and Gamma distributions (Pearson Type III). The mode of the Pearson distribution is at the point $x = a$.

Pearson derived thirteen distribution types representing the continuous distributions of the statistical theory. These are:

Type I:

$$g(x) = k \left(1 + \frac{x}{a_1} \right)^{m_1} \left(1 - \frac{x}{a_2} \right)^{m_2} \quad (3.109)$$

$$-a_1 \leq x \leq a_2 \quad (3.110)$$

$$m_1, m_2 > -1 \quad (3.111)$$

This is the Beta distribution.

Type II:

$$g(x) = k \left(1 - \frac{x^2}{a^2}\right)^m \quad (3.112)$$

$$-a \leq x \leq a \quad (3.113)$$

$$m \geq -1 \quad (3.114)$$

The Rectangular distribution is a special case of Type II.

Type III:

$$g(x) = k \left(1 + \frac{x}{a}\right)^{\gamma a} e^{-\gamma x} \quad (3.115)$$

$$-a \leq x \leq \infty \quad (3.116)$$

$$\gamma > 0 \quad (3.117)$$

$$a > 0 \quad (3.118)$$

This is the Gamma distribution.

Type IV:

$$g(x) = k \left(1 + \frac{x^2}{a^2}\right)^{-m} e^{-\mu \tan^{-1} \left(\frac{x}{a}\right)} \quad (3.119)$$

$$-\infty \leq x \leq \infty \quad (3.120)$$

$$a > 0 \quad (3.121)$$

$$\mu > 0 \quad (3.122)$$

Type V:

$$g(x) = k x^{-p} e^{-r/x} \quad (3.123)$$

$$0 \leq x \leq \infty \quad (3.124)$$

$$r > 0 \quad (3.125)$$

$$p > 1 \quad (3.126)$$

TYPE VI:

$$g(x) = k x^{-q_1} (x - a)^{q_2} \quad (3.127)$$

$$a \leq x \leq \infty \quad (3.128)$$

$$q_1 > q_2 - 1 \quad (3.129)$$

Type VII:

$$g(x) = k \left(1 + \frac{x^2}{a^2}\right)^{-m} \quad (3.130)$$

$$-\infty \leq x \leq \infty \quad (3.131)$$

$$m > \frac{1}{2} \quad (3.132)$$

The t-distribution is a special case of Type VII.

Type VIII:

$$g(x) = k \left(1 + \frac{x}{a}\right)^{-m} \quad (3.133)$$

$$-a \leq x \leq 0 \quad (3.134)$$

$$0 \leq m \leq 1 \quad (3.135)$$

Type IX:

$$g(x) = k \left(1 + \frac{x}{a}\right)^m \quad (3.136)$$

$$-a \leq x \leq 0 \quad (3.137)$$

$$m > -1 \quad (3.138)$$

Type X:

$$g(x) = \lambda e^{-\lambda x} \quad (3.139)$$

$$x > 0 \quad (3.140)$$

$$\lambda > 0 \quad (3.141)$$

This is the exponential distribution.

Type XI:

$$g(x) = k x^{-m} \quad (3.142)$$

$$b \leq x \leq \infty \quad (3.143)$$

$$m > 0 \quad (3.144)$$

Type XII:

$$g(x) = \left[\left(1 + \frac{x}{a_1}\right) / \left(1 - \frac{x}{a_2}\right) \right]^m \quad (3.145)$$

$$-a_1 \leq x \leq a_2 \quad (3.146)$$

$$m > 1 \quad (3.147)$$

Type XIII:

$$g(x) = k e^{-x^2/2\sigma^2} \quad (3.148)$$

$$-\infty \leq x \leq \infty \quad (3.149)$$

$$\sigma > 0 \quad (3.150)$$

This is the normal distribution.

The Pearson distribution type is classified according to the following Pearson parameters as measured from the data's moments:

$$\beta_1 = \frac{m_3^2}{m_2^3} \quad (3.151)$$

$$\alpha_3 = \sqrt{\beta_1} \quad (3.152)$$

$$\beta_2 = \frac{m_4}{m_2^2} \quad (3.153)$$

$$\alpha_4 = \beta_2 \quad (3.154)$$

$$K = \frac{\beta_1(\beta_2 + 3)^2}{4(4\beta_2 - 3\beta_1)(2\beta_2 - 3\beta_1 - 6)} \quad (3.155)$$

$$\lambda = \frac{(4\beta_2 - 3\beta_1)(10\beta_2 - 12\beta_1 - 18)^2 - \beta_1(\beta_2 + 3)^2(8\beta_2 - 9\beta_1 - 12)}{(3\beta_1 - 2\beta_2 + 6)[\beta_1(\beta_2 + 3)^2 + 4(4\beta_2 - 3\beta_1)(3\beta_1 - 2\beta_2 + 6)]} \quad (3.156)$$

where

m_i = sample moment,

α_3 = measures the skewness of the distribution relative to its degree of spread, called third standard moment,

β_2 = relative measure of Kurtosis, and

$\alpha_4 = \beta_2$ = fourth standard moment.

The following criteria are used for classifying a Pearson distribution:

<u>Distribution Type</u>	<u>Criteria</u>
I	$k < 0$
II	$k = 0, \beta_1 = 0, \beta_2 < 3$
III	$2\beta_2 - 3\beta_1 - 6 = 0$
IV	$0 < k < 1$
V	$k = 1$
VI	$k > 1$
VII	$k = 0, \beta_1 = 0, \beta_2 > 3$
VIII	$k < 0, \lambda = 0, 5\beta_2 - 6\beta_1 - 9 < 0$
IX	$k < 0, \lambda = 0, 5\beta_2 - 6\beta_1 - 9 > 0, 2\beta_2 - 3\beta_1 - 6 < 0$
X	$\beta_1 = 4, \beta_2 = 9$
XI	$k = 1, \lambda = 0, 2\beta_2 - 3\beta_1 > 0$
XII	$5\beta_2 - 6\beta_1 - 9 = 0$
XIII	$k = 0, \beta_1 = 0, \beta_2 = 3$

3.4.4 Applications of Statistical Distributions

The following charts are extracted from Hahn and Shapiro [25] (by permission) and give applications of different statistical distributions.

Distribution	Application	Example	Comments
Normal	A basic distribution of statistics. Many applications arise from central limit theorem (average of values of n observations approaches normal distribution, irrespective of form of original distribution under quite general conditions). Consequently, appropriate model for many—but not all—physical phenomena.	Distribution of physical measurements on living organisms, intelligence test scores, product dimensions, average temperatures, and so on.	Tabulation of cumulative values of standardized normal distribution readily available. Many methods of statistical analysis presume normal distribution.
Gamma	A basic distribution of statistics for variables bounded at one side—for example, $0 \leq x < \infty$. Gives distribution of time required for exactly k independent events to occur, assuming events take place at a constant rate. Used frequently in queueing theory, reliability, and other industrial applications.	Distribution of time between recalibrations of instrument that needs recalibration after k uses; time between inventory restocking, time to failure for a system with standby components.	Cumulative distribution values have been tabulated. Erlangian, exponential, and chi-square distributions are special cases.
Exponential	Gives distribution of time between independent events occurring at a constant rate. Equivalently, probability distribution of life, presuming constant conditional failure (or hazard) rate. Consequently, applicable in many—but not all—reliability situations.	Distribution of time between arrival of particles at a counter. Also life distribution of complex nonredundant systems, and usage life of some components—in particular, when these are exposed to initial burn-in, and preventive maintenance eliminates parts before wear-out.	Special case of both Weibull and gamma distributions.
Beta	A basic distribution of statistics for variables bounded at both sides—for example $0 \leq x \leq 1$. Useful for both theoretical and applied problems in many areas.	Distribution of proportion of population located between lowest and highest value in sample; distribution of daily per cent yield in a manufacturing process; description of elapsed times to task completion (PERT).	Cumulative distribution values have been tabulated. Uniform, right triangular, and parabolic distributions are special cases.
Uniform	Gives probability that observation will occur within a particular interval when probability of occurrence within that interval is directly proportional to interval length.	Used to generate random values.	Special case of beta distribution.
Log-normal	Permits representation of random variable whose logarithm follows normal distribution. Model for a process arising from many small multiplicative errors. Appropriate when the value of an observed variable is a random proportion of the previously observed value.	Distribution of sizes from a breakage process; distribution of income size, inheritances and bank deposits; distribution of various biological phenomena; life distribution of some transistor types.	
Rayleigh	Gives distribution of radial error when the errors in two mutually perpendicular axes are independent and normally distributed around zero with equal variances.	Bomb-sighting problems; amplitude of noise envelope when a linear detector is used.	Special case of Weibull distribution.
Cauchy	Gives distribution of ratio of two independent standardized normal variates.	Distribution of ratio of standardized noise readings; distribution of $\tan \theta$ when θ is uniformly distributed.	Has no moments.
Weibull	General time-to-failure distribution due to wide diversity of hazard-rate curves, and extreme-value distribution for minimum of N values from distribution bounded at left.	Life distribution for some capacitors, ball bearings, relays, and so on.	Rayleigh and exponential distributions are special cases.
Extreme value	Limiting model for the distribution of the maximum or minimum of N values selected from an "exponential-type" distribution, such as the normal, gamma, or exponential.	Distribution of breaking strength of some materials, capacitor breakdown voltage, gust velocities encountered by airplanes, bacteria extinction times.	Cumulative distribution has been tabulated.

Figure 1. Summary: Applications of Continuous Statistical Distributions

Distribution	Application	Example	Comments
Binomial	Gives probability of exactly x successes in n independent trials, when probability of success p on <i>single</i> trial is a constant. Used frequently in quality control, reliability, survey sampling, and other industrial problems.	What is the probability of 7 or more "heads" in 10 tosses of a fair coin?	Can sometimes be approximated by normal or by Poisson distribution.
Multinomial	Gives probability of exactly x_i outcomes of event i , for $i = 1, 2, \dots, k$ in n independent trials when the probability p_i of event i in a <i>single</i> trial is a constant. Used frequently in quality control and other industrial problems.	Four companies are bidding for each of three contracts, with specified success probabilities. What is the probability that a single company will receive all the orders?	Generalization of binomial distribution for more than 2 outcomes.
Hypergeometric	Gives probability of picking exactly x good units in a sample of n units from a population of N units when there are k bad units in the population. Used in quality control and related applications.	Given a lot with 21 good units and four defectives. What is the probability that a sample of five will yield not more than one defective?	May be approximated by binomial distribution when n is small relative to N .
Geometric	Gives probability of requiring exactly x binomial trials before the first success is achieved. Used in quality control, reliability, and other industrial situations.	Determination of probability of requiring exactly five test firings before first success is achieved.	
Pascal	Gives probability of exactly x failures preceding the s th success.	What is the probability that the third success takes place on the 10th trial?	
Negative Binomial	Gives probability similar to Poisson distribution (see below) when events do not occur at a constant rate and occurrence rate is a random variable that follows a gamma distribution.	Distribution of number of cavities for a group of dental patients.	Generalization of Pascal distribution when s is not an integer. Many authors do not distinguish between Pascal and negative binomial distributions.
Poisson	Gives probability of exactly x independent occurrences during a given period of time if events take place independently and at a constant rate. May also represent number of occurrences over constant areas or volumes. Used frequently in quality control, reliability, queueing theory, and so on.	Used to represent distribution of number of defects in a piece of material, customer arrivals, insurance claims, incoming telephone calls, alpha particles emitted, and so on.	Frequently used as approximation to binomial distribution.

Figure 2. Summary: Applications of Discrete Statistical Distributions

CHAPTER IV

SYSTEM CONSTRUCTION AND VALIDATION

4.1 Preliminary Considerations

In general, a risk analysis problem is too complex to lend itself to an analytical model. The model developed here uses Monte Carlo simulation technique and FORTRAN IV. Simulation, with the availability of high speed computers and low cost of computation, has become an effective technique in solving problems where analytical solutions are untenable. Monte Carlo technique can be easily understood and imposes no restrictions. FORTRAN IV is familiar to most analysts and its compilers are standard software in most computer installations. These were important factors in the choice of the simulation model presented here.

Flexibility of the system is also an important factor, especially in our case here where wide variations exist among possible investment alternatives. Modular design is therefore a major feature of UPFAR, the system presented here. Another necessary flexibility that was taken into consideration is the ease of interaction among the different modules of the system.

4.2 UPFAR: Utility Program For Analysis of Risk -- Basic System Structure

The UPFAR package consists of four major modules serving the four basic functions in a risk analysis study:

- 1) Probability distribution fitting: Empirical data that are compiled from historical experiences are run through the program PRODIP (Probability Distribution Program) to get the best fit of a predecided distribution or, to get the best probability distribution that fits the data.
- 2) Simulation: GUSS (General Utility Simulation System) samples from the different distributions, links with the problem's model, and performs housekeeping operations for inputs and outputs.
- 3) Investment model: INMO (Investment Model) includes the different mathematical relationships between the variables of the problem under study.
- 4) Utility Evaluation and Measurement: RUM (Routine for Utility Measurement) translates the probability distributions of the output parameters into utility measures.

The functional relationships between the UPFAR modules is given in Figure 3. In general, a risk analysis study goes through the following procedure:

- 1) Collect data related to the different parameters to the problem.
- 2) Run PRODIP on these data to determine the best probability distribution that fits these data.
- 3) Enter probabilities obtained from PRODIP, together with other probabilities that may be estimated on a subjective basis

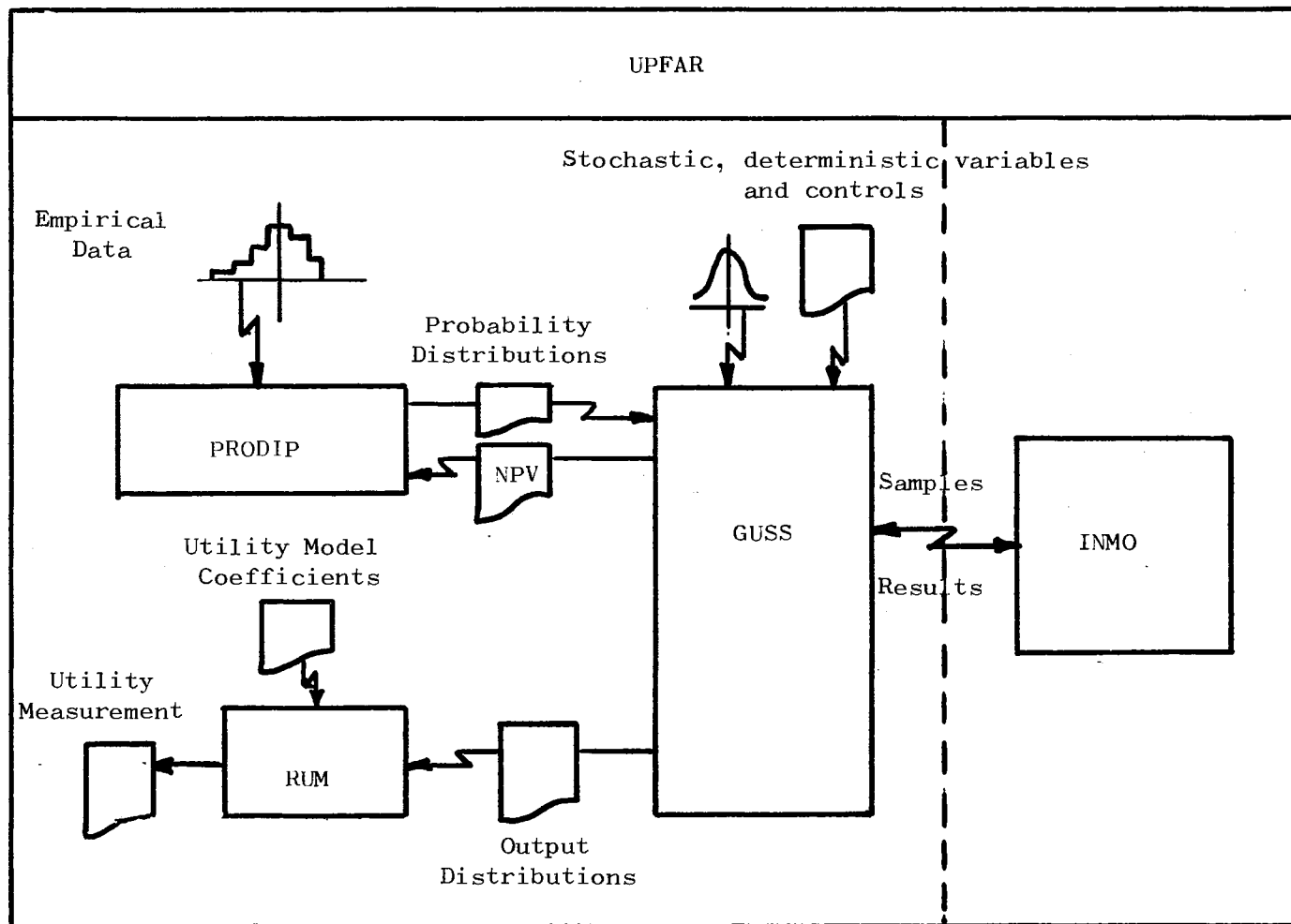


Figure 3. Utility Program For Analysis of Risk (UPFAR)--Flow Chart

into GUSS.

- 5) Run GUSS together with INMO (or any other special subroutine representing the relationships between the investment parameters) for simulation.
- 6) Enter the results obtained from GUSS, in the form of a frequency distribution of the investment's NPV into PRODIP to determine the best fit for these output data.
- 7) Run the distribution parameters of the NPV, as obtained in step 6, in RUM to obtain a measure of the investment's utility.
- 8) Make final decisions of whether or not to accept the investment on the basis of the above results and other relevant information.

4.2.1 PRODIP: Probability Distribution

Fitting Program

PRODIP is a program that tries to find the best fit from a family of statistical distributions to historical or empirical data. PRODIP is a modified version of a UNIVAC 1108 program that was developed by NASA [24].

PRODIP offers two options, one for fitting continuous distributions to the data, and the other option fits discrete distributions (see Figure 4).

Option A: This option of PRODIP tries to find the best fit out of five of the Pearson family of distributions mentioned in section 3.4.3. These five distributions are Pearson distributions types I, III, IV, VI, and XIII based on the different moments of the data.

Option B: This option of PRODIP tries to fit empirical data to one or more of the following seven continuous distributions, discussed in section 3.4.2, as requested:

- 1) Exponential
- 2) Normal
- 3) Lognormal
- 4) Fisher-Tippett I
- 5) Fisher-Tippett II
- 6) Weibull
- 7) Beta

Option C: This option of PRODIP tries to fit empirical data to one or more of the following five discrete distributions discussed in section 3.4.2 as requested:

- 1) Poisson
- 2) Truncated Poisson
- 3) Binomial
- 4) Negative Binomial
- 5) Truncated Negative Binomial

While PRODIP offers these options to the users, it should be reminded that the choice of any given distribution to the data should be dependent on the nature of the variable and the physical properties of the distributions. For example, if the variable can only assume discrete values, it would be inappropriate to fit to it any of the continuous distributions that are offered in Option B, or a Pearson distribution, offered in Option A, which is also continuous. On the other hand, if the variable can be satisfied with any one of the continuous distributions offered in Option B, one would then select the

distribution that best fits the data as measured by the χ^2 or the Kolmogorov-Smirnov tests provided in the program.

4.2.2 GUSS: General Utility Simulation System

GUSS is a general utility simulation system that uses Monte Carlo technique. GUSS is designed in a way that makes it easy to use as a general purpose simulator for risk analysis studies as well as other simulation problems. GUSS has the following responsibilities:

- 1) Accepts input data about the problem's constants, deterministic variables, and probability distributions of the stochastic variables as well as sorting and reporting controls.
- 2) Generates samples from the different probability distributions.
- 3) Links with INMO to calculate the resultant NPV (or other parameters that are of interest) of the generated sample. Each time the link is made with INMO is a one simulation iteration.
- 4) Repeats steps 2 and 3 until the total number of desired iterations as given to the program have been completed.
- 5) Sorts the output results of the simulation and orders them in a descending order, giving the probability distribution of the output NPV measure.
- 6) Calculates different statistics about the output distribution as well as a distribution summary.

GUSS is characterized by its simple linkage with a subroutine representing the model of the investment problem (or any other simulation problem). Figure 5 gives a flow diagram of GUSS.

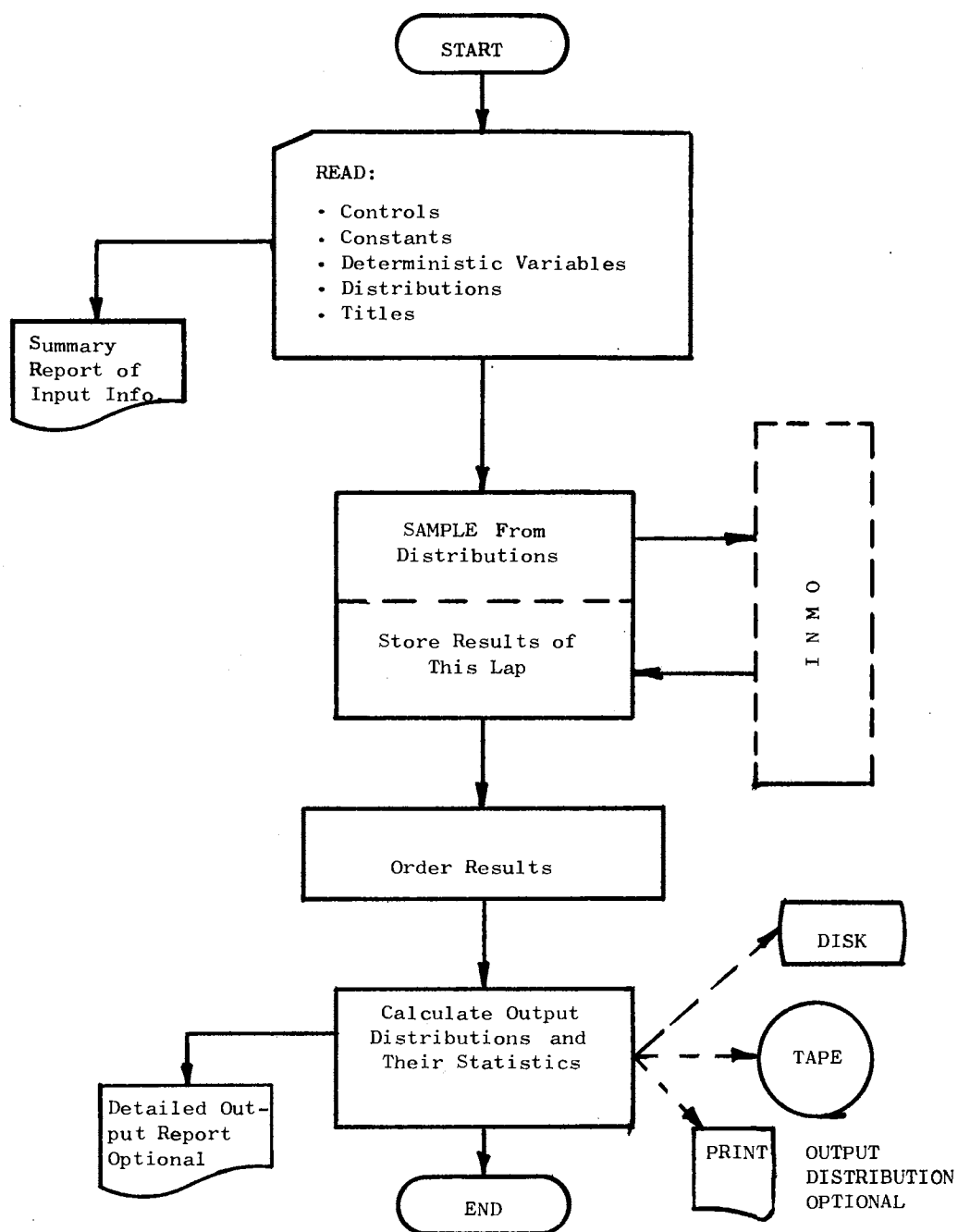


Figure 5. General Utility Simulation System (GUSS -- Flow Chart

Guss has the capability of handling simulation problems having the following characteristics:

- Up to 50 integer constants.
- Up to 50 deterministic variables.
- Up to 50 stochastic variables, continuous or discrete.
- Up to 25 testing and acceptability criteria.
- Up to 10 counters.
- Up to 7 output parameters.
- Up to 10 reports with 40 footnotes each, 20 of these footnotes may have values calculated in the simulation.
- Up to 1000 simulation iterations.
- Up to 10 output reports on the simulation results.
- Up to 10 sorts per output report.
- Up to 5 output devices, disks and/or tapes for output parameters detailed distributions.

4.2.3 INMO: Investment Model

The variations between different investment problems is so great that no one model can encompass all these variations. This was a major factor in designing INMO as an independent module (a subroutine) that links with GUSS through general names for the input and output variables.

The INMO model presented here is applicable to a wide range of investment problems that may have any subset or all of the following variables:

- 1) Capital Investments:
 - a) Working Capital.
 - b) Land.
 - c) Equipment and other depreciable investment items.

The following is allowed in the model:

- a) Investment items can be owned or leased.
 - b) A time lag can be assumed before the investment item is installed.
 - c) INMO automatically replaces items that retire during the life of the project with the same items.
 - d) Three depreciation methods are available: Straight Line, Sum of the Years Digits, and Double Rate Declining Balance.
- 2) Sales and Revenues:
- The following is allowed for each product:
- a) A time lag until the product is introduced to the market.
 - b) A growth period to maturity volumes (linear growth rate).
 - c) A price per unit for each product.
 - d) A direct cost per unit for each product.
- 3) Overhead Expenses:
- The following is allowed for each expense item:
- a) A time lag until the expense is to be incurred.
 - b) A growth period to maturity of overhead.
- 4) Tax Rate:
- The calculations of cash flows are made on an after-tax basis.
- 5) Net Present Value:
- The program can calculate and present the distributions of up to seven NPV's calculated for up to seven rate of return assumptions.

Modifications can be made to the program to enable it to handle variations to the above. The degree of ease or difficulty of such changes are dependent on the variance of the characteristics of the new problem to what the present INMO model can handle. For example,

INMO can be easily expanded to allow a growth-function of the maturity volumes, etc. Figure 6 gives a flow diagram of INMO.

4.2.4 RUM: Routine for Utility Measurement

This module includes different utility models presented in section 3.3. The utility models included are:

- 1) Farrer model, section 3.3.5.
- 2) Cramer's and Smith's model, section 3.3.6.
- 3) Bussey's model, section 3.3.7.

Data about the NPV distribution statistics are entered to RUM for evaluation and making investment decisions. The constants and parameters of the desired utility model are supplied by the user. Figure 7 gives a flow diagram of RUM.

4.3 System Validation

Validation of UPFAR was performed at two different levels:

- 1) Validation of each module separately to ascertain the module is error free.
- 2) Validation of the integrity of the System as a whole to ensure the proper functioning of the whole system and that no interaction problems would be encountered.

The first level of validation was accomplished during the development of the module. The different situations that the module would encounter were enumerated and tested by comparing computer runs against hand calculations.

The second level of validation was performed by trial runs of different possible interactions between the four modules and comparing

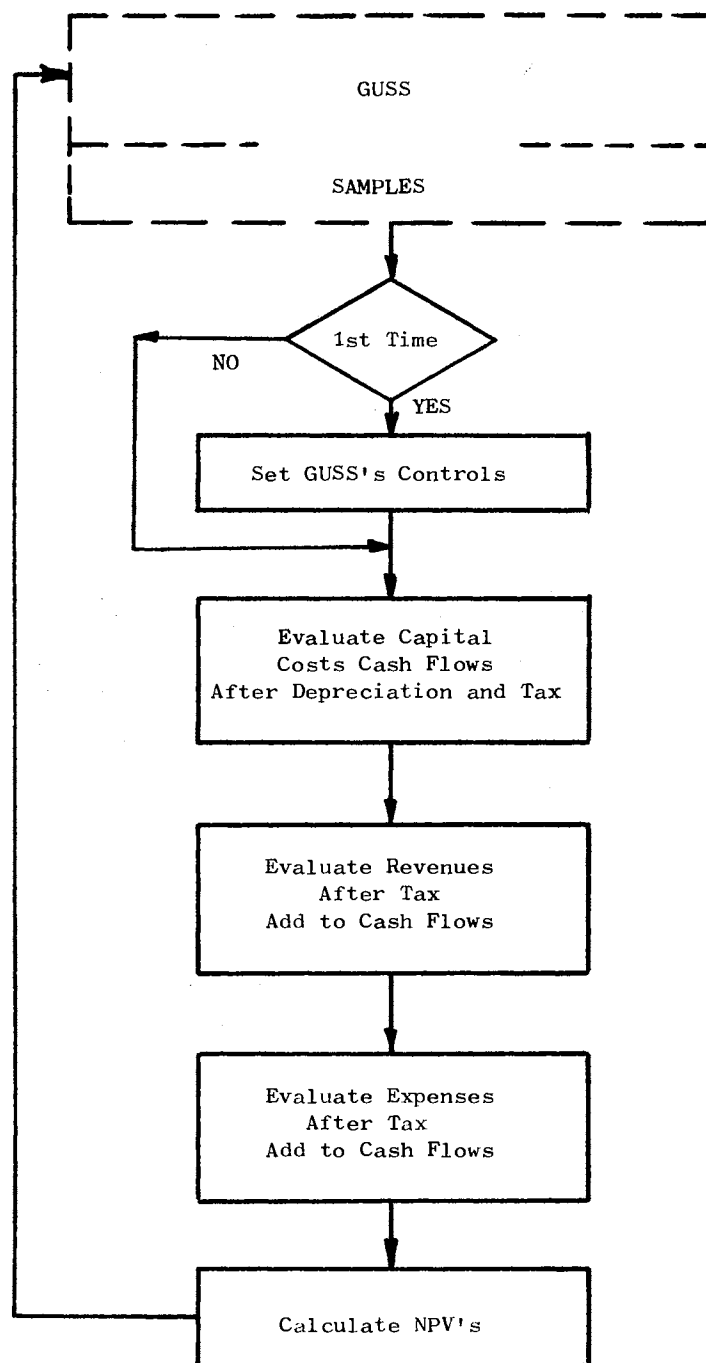


Figure 6. Investment Model (INMO) --
Flow Chart

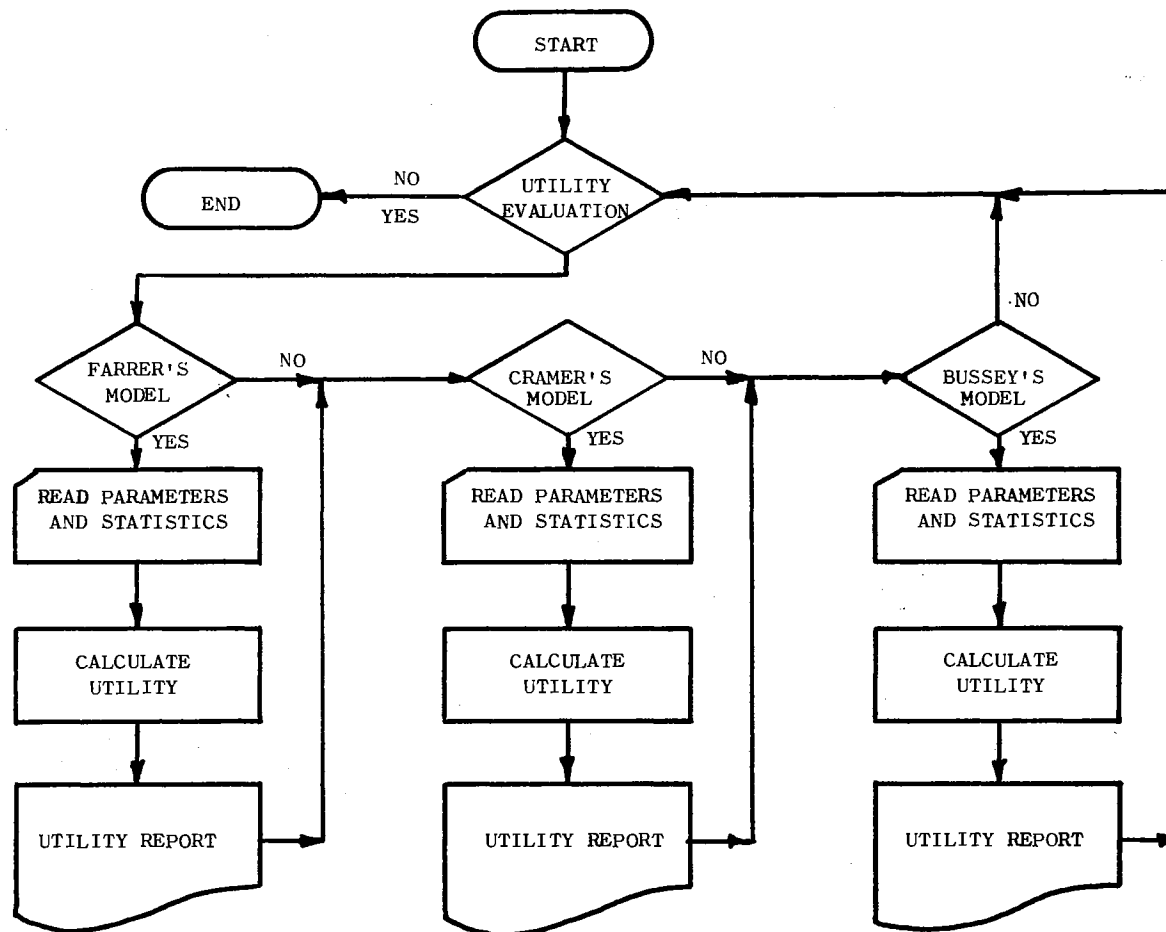


Figure 7. Routine for Utility Measurement (RUM)--Flow Chart

the results with hand calculations. A comment should be made, however, that no matter how strong the claims about the thoroughness of the validation tests, a 100% error free system may not be guaranteed. Remote and peculiar errors, for practical reasons, would be corrected if encountered in the future.

4.4 Notes on UPFAR

The use of the UPFAR system is a simple task as a result of its modular design, flexibility, and ease of input/output procedures. The major attention in a risk analysis or other simulation problems should be directed to the conceptual aspects of the problem under study while preparing the input, choosing certain options of the model, and to the analysis of the output. Verification that a chosen distribution, the INMO model, and the chosen utility model, do represent the real life problem is essential. Otherwise, the output results, regardless of how good they look, become of no value.

To use the UPFAR modules, the following procedure is recommended:

- 1) Identify the different variables and parameters that are pertinent to the problem.
- 2) Ascertain that the input data are representative of the variable under study and that no extraneous observations or extreme cases are included.
- 3) Identify the characteristics of the variable under study and ascertain that the variable is not being misrepresented by a chosen distribution.
- 4) Validate that the resultant distribution that best fits the data agrees with real life situations.

data agrees with real life situations.

- 5) Ensure that you are using the appropriate INMO subroutine and that your problem is being represented by that model.
- 6) Choose the number of simulation iterations that would provide the desired accuracy.
- 7) Ascertain that the chosen utility model and its parameters represent the investor's attitudes.
- 8) When presenting the results of the study, make the decision maker aware that:
 - a) There is no assurance that the outcome of a recommended decision is guaranteed.
 - b) Risk analysis does not eliminate uncertainty. It rather leads to a clearer understanding of the result of the interaction between the decision elements.

CHAPTER V

SAMPLE CASE STUDY -- ANALYSIS OF INVESTMENT IN OUTLET DEPOTS

The purpose of this chapter is to illustrate how to use UPFAR in the analysis of investments involving risk using a sample case study. The data used for this case study are for illustrative purpose.

5.1 The INCO Company Investment Problem

The INCO (Investment Company) company has an investment prospective to enter a given market with its two major products: Prodone and Prodtwo. An an alternative to this investment, INCO can lend resources to its subsidiaries at a 10% return.

5.2 Analysis of the Investment Opportunity -- Data Collection

INCO starts getting the information necessary to analyze the investment opportunity. This information is the subjective feelings of experts in the field, results from analyzing empirical data, using PRODIP, or are the results of similar experiences. This information is given as follows:

1) Capital Items

- a) Land: INCO has an option to lease the land required for the depot. The leasing company is asking for a leasing

cost of \$20,000 per year on a twenty year lease basis. However, INCO feels that they may be able to negotiate a lower lease. The experts at INCO, after some discussion, felt that the probabilities given in Table I represent the chances of different lease values. If the investment is accepted, the land will be leased at the start of the investment life to allow for building construction and installations.

- b) Working Capital: After the land has been leased, working capital will be needed during the two phases of the project, its build-up stage, and its operational period. INCO accordingly assumes that the working capital will be recovered after the twenty-year period of project life. From INCO's previous experiences, the distribution of working capital was constructed as given in Table II.
- c) Buildings: After the land has been leased, building constructions are to start and the buildings can be assumed to be ready in two years. INCO would have to pay the cost of the buildings after one year from the start. The distribution of the investments in buildings, based on INCO's previous experiences, is given in Table III.
- d) Equipment: As for equipment, the distributions of the investments in heavy and light equipment are given in Tables IV and V, respectively. The service life, the salvage value, and the depreciation methods, as allowed by tax regulations, are given in Table VI for each of the capital items.

TABLE I
LEASE VALUE FOR LAND NECESSARY FOR
THE INCO COMPANY PROJECT

Lease Value \$	Cumulative Probability % for \leq Value	Probability %
12,000	0	0
15,000	20	20
18,000	70	50
20,000	100	30

TABLE II
WORKING CAPITAL NEEDED FOR THE
INCO COMPANY PROJECT

Working Capital \$	Cumulative Probability % for \leq Value	Probability %
50,000	0	0
60,000	30	30
70,000	75	45
80,000	90	15
90,000	100	10
100,000	100	0

TABLE III
INVESTMENTS IN BUILDINGS NEEDED
FOR THE INCO COMPANY PROJECT

Building Value \$	Cumulative Probability % for \leq Value	Probability %
10,000	0	0
12,000	50	20
14,000	70	50
16,000	85	15
18,000	100	15

TABLE IV
INVESTMENT IN HEAVY EQUIPMENT NEEDED
FOR THE INCO COMPANY PROJECT

Heavy Equipment Value \$	Cumulative Probability % for \leq Value	Probability %
5,000	0	0
10,000	5	5
20,000	25	20
30,000	75	50
40,000	90	15
50,000	100	10

TABLE V
INVESTMENT IN LIGHT EQUIPMENT NEEDED
FOR THE INCO COMPANY PROJECT

Light Equipment Value \$	Cumulative Probability % for \leq Value	Probability %
5,000	0	0
10,000	5	5
20,000	15	10
25,000	25	10
30,000	85	60
40,000	95	10
50,000	100	5

TABLE VI
ALLOWED DEPRECIATION FOR INVESTMENT ITEMS

Investment Item	Service Life	% Salvage	Depreciation Method
Buildings	25	30	Sum of the Years Digits
Heavy Equipment	10	20	Double Rate Declining Balance
Light Equipment	5	10	Straight Line

2) Sales and Gross Margins

Based on INCO's marketing strategy, INCO expects to start selling Prodone at the beginning of the third year from the start. INCO expects to introduce Prodtwo at the beginning of the fourth year. However, these target dates are subject to the completion of construction and equipment installation. Accordingly, the probability of the starting time to sell each of the two products is given in Tables VII and VIII, based on INCO's experience. After a market study of the location, INCO's marketing experts also provided the distributions of the maturity volumes for each of the two products as well as the growth period required for their maturities. These are given in Tables IX through XII. As for prices and direct costs, these are estimated based on economic forecasts by the economic research department as well as considerations from the marketing and technical groups. These are given in Tables XIII through XVI.

3) Overhead Expenses

INCO classifies its overhead expenses into two categories: advertising and fixed expenses. Based on INCO's estimates, overhead expenses would start with the opening of the depot after the second year and would grow over the next period of two years until it reaches maturity. Advertising expenses would start one year before the first product is introduced. It would grow over the next five years until it reaches its maturity value. The distributions of the fixed and advertising expenses and their scheduling is given in Tables XVII through XXII.

TABLE VII
START OF SELLING FOR PRODUCT PRODONE

Time From Start Years	Cumulative Probability % for \leq Value	Probability %
1.50	0	0
1.75	50	50
2.00	75	25
2.25	85	10
2.50	95	10
2.75	100	5

TABLE VII
START OF SELLING FOR PRODUCT PRODTWO

Time From Start Years	Cumulative Probability % for \leq Value	Probability %
2.25	0	0
2.50	20	20
2.75	45	25
3.00	70	25
3.50	95	25
3.75	100	5

TABLE IX
MATURITY SALES VOLUME FOR PRODUCT PRODONE

Sales Volume \$	Cumulative Probability % for \leq Value	Probability %
50,000	0	0
60,000	10	10
70,000	20	10
80,000	40	20
90,000	90	50
100,000	100	10

TABLE X
MATURITY SALES VOLUME FOR PRODUCT PRODTWO

Sales Volume \$	Cumulative Probability % for \leq Value	Probability %
20,000	0	0
25,000	10	10
30,000	60	50
40,000	80	20
50,000	90	10
60,000	100	10

TABLE XI
GROWTH PERIOD TO MATURITY FOR PRODUCT PRODONE

Growth Period Years	Cumulative Probability % for \leq Value	Probability %
0.50	0	0
1.00	20	20
1.50	60	40
2.00	90	30
2.50	100	10

TABLE XII
GROWTH PERIOD TO MATURITY FOR PRODUCT PRODTWO

Growth Period Years	Cumulative Probability % for \leq Value	Probability %
2.25	0	0
2.50	25	25
2.75	50	25
3.00	75	25
3.25	100	25

TABLE XIII
SELLING PRICE PER UNIT OF PRODONE

Selling Price \$	Cumulative Probability % for \leq Value	Probability %
2.20	0	0
2.50	10	10
2.70	30	20
3.00	70	40
3.20	90	20
3.50	100	10

TABLE XIV
SELLING PRICE PER UNIT OF PRODTWO

Selling Price \$	Cumulative Probability % for \leq Value	Probability %
2.50	0	0
2.75	10	10
3.00	30	20
3.25	70	40
3.50	90	20
4.00	100	10

TABLE XV
DIRECT COST PER UNIT OF PRODONE

Direct Cost Per Unit \$	Cumulative Probability % for \leq Value	Probability %
1.50	0	0
1.70	15	15
1.90	55	40
2.10	75	20
2.30	95	20
2.50	100	5

TABLE XVI
DIRECT COST PER UNIT OF PRODTWO

Direct Cost Per Unit \$	Cumulative Probability % for \leq Value	Probability %
1.70	0	0
1.80	15	15
2.00	40	25
2.20	60	20
2.50	70	10
2.75	90	20
3.00	100	10

TABLE XVII
MATURITY LEVEL OF FIXED EXPENSES PER YEAR
FOR THE INCO PROJECT

Fixed Expenses \$	Cumulative Probability % for \leq Value	Probability %
20,000	0	0
22,000	10	10
24,000	20	10
26,000	60	40
28,000	80	20
30,000	85	5
32,000	90	5
34,000	95	5
36,000	100	5

TABLE XVIII
MATURITY LEVEL OF ADVERTISING EXPENSES PER YEAR
FOR THE INCO PROJECT

Advertising Expenses \$	Cumulative Probability % for \leq Value	Probability %
8,000	0	0
9,000	50	50
10,000	70	20
11,000	90	20
12,000	95	5
15,000	100	5

TABLE XIX
STARTING TIME FOR FIXED EXPENSES

Start of Expense Year	Cumulative Probability % for \leq Value	Probability %
0.50	0	0
1.00	10	10
1.25	60	50
1.50	80	20
2.00	90	10
2.50	95	5
3.00	100	5

TABLE XX
STARTING TIME FOR ADVERTISING EXPENSES

Start of Expense Year	Cumulative Probability % for \leq Value	Probability %
1.25	0	0
1.50	5	5
1.75	55	50
1.90	70	25
2.00	90	15
2.20	95	5
2.50	100	5

TABLE XXI
GROWTH PERIOD TO MATURITY FOR FIXED EXPENSES

Growth Period Years	Cumulative Probability % for \leq Value	Probability %
1.00	0	0
1.50	50	50
2.00	75	25
2.50	85	10
3.00	90	5
3.50	95	5
4.00	100	5

TABLE XXII
GROWTH PERIOD TO MATURITY FOR ADVERTISING EXPENSES

Growth Period Years	Cumulative Probability % for \leq Value	Probability %
2.75	0	0
3.00	15	15
3.50	40	25
4.00	60	20
4.50	85	15
5.00	90	5
5.50	95	5
6.00	100	5

4) Tax

The distribution of the tax rates are again estimated by the economic research department and is given in Table XXIII.

5.3 Preparation of Input for INMO

The information given in Tables I through XXIII is encoded on the forms for INMO's data sheets given in Section A.3 and following the instructions in that section. The filled forms are given in Appendix B, pp. 182-199. After data key punching, the investment is simulated in INMO, giving the results shown in Appendix B, pp. 200-213.

5.4 Preparing the Information for an

UPFAR Run

The information collected about the different investment parameters is encoded on INMO data sheets following the procedures given in Section A.3.2. For the input distributions, interpolation is made to arrive to the five percentile points of the distribution where missing. The input sheets for the UPFAR run are given in Appendix B, pp. 182-199.

The competitive alternative to our prospective investment, the lending to subsidiaries, offers a return of 10%. Using this rate and not considering other factors such as intangibles, for example, for the comparison between these two alternatives, is normally sufficient. However, different rates, in addition to the competitor's may be included. This may be sometimes favorable since it allows answers to "what if" conditions that may be asked when presenting the results. Furthermore, including other rates lower and higher than the one we are to use, would help answer the question of "what is the rate of return on

TABLE XXIII

TAX RATE

% Tax	Cumulative Probability %% for \leq Value	Probability %
40	0	0
45	10	10
50	70	60
55	90	20
60	95	5
65	100	5

the project?" For the INCO company case, six additional interest rates were used in addition to the 10%. These rates are 5%, 9%, 12%, 15%, 20%, and 25%.

For the number of simulation iterations, 200 iterations is chosen for practical considerations of accuracy and computer cost. The detailed distribution output is also requested on the printer for analysis for possible run on PRODIP.

One output report is requested for this study. This output report gives the distribution of the NPV for the seven discounting rates. The distribution for the NPV of any of the discounting rates is obtained by sorting its output column and printing the entire column. This explains the output report's control cards.

5.5 Analysis of the Simulation Results

From the output report, Appendix B, pp. 207-213, one concludes the following results:

- 1) For the 10% rate:
 - a) A net present value of \$47,477 gain is expected from the investment with a standard deviation of \$139,140.
 - b) There is a probability of 37.5% that the project incurs a loss.
 - c) The highest probable gain in NPV is \$535,500 and the lowest probable NPV loss is \$326,030.
 - d) The expected gain is approximately \$86,600 and the expected loss is \$36,500 (calculated from the distribution of NPV, pp. 209-212).

The above information provides the following additional measures, see section 3.2.1, pp. 12-13:

- a) Variance Ratio = $\sigma(V)/E(V) = 2.93$.
- b) Loss Ratio = $E(L)/E(V) = 0.77$.
- c) Gain Ratio = $E(G)/E(V) = 1.82$.
- d) Odds on gain = $\frac{\text{probability of gain}}{\text{probability of loss}} = 1.66$.

5.6 Distribution of the Net Present Value of the Investment at 10% Rate

A diagram of the probability distribution of the NPV of the investment at a 10% discounting rate is given in Figure 8. Having the data for the detailed distribution, it is interesting to run them in the PRODIP program. PRODIP would try to select the best fit to the data out of the Pearson system of distributions as well as standard statistical distributions. Furthermore, PRODIP gives the different moments, necessary for the different utility models.

To analyze the NPV distribution data on PRODIP, a change of scale and transfer of axes is necessary to maintain the positivity conditions on the data required for some of the statistical distribution. The change of scale is necessary to eliminate an overflow or an underflow that may occur when using exponents in some distributions. For this reason, a value of \$500,000 is added to each observation to eliminate negative observations. Observations are also multiplied by 10^{-5} to eliminate overflows.

The input to PRODIP is given in Appendix B, pp. 214-216, and the results of running PRODIP on the data is given in Appendix B, pp. 217-230

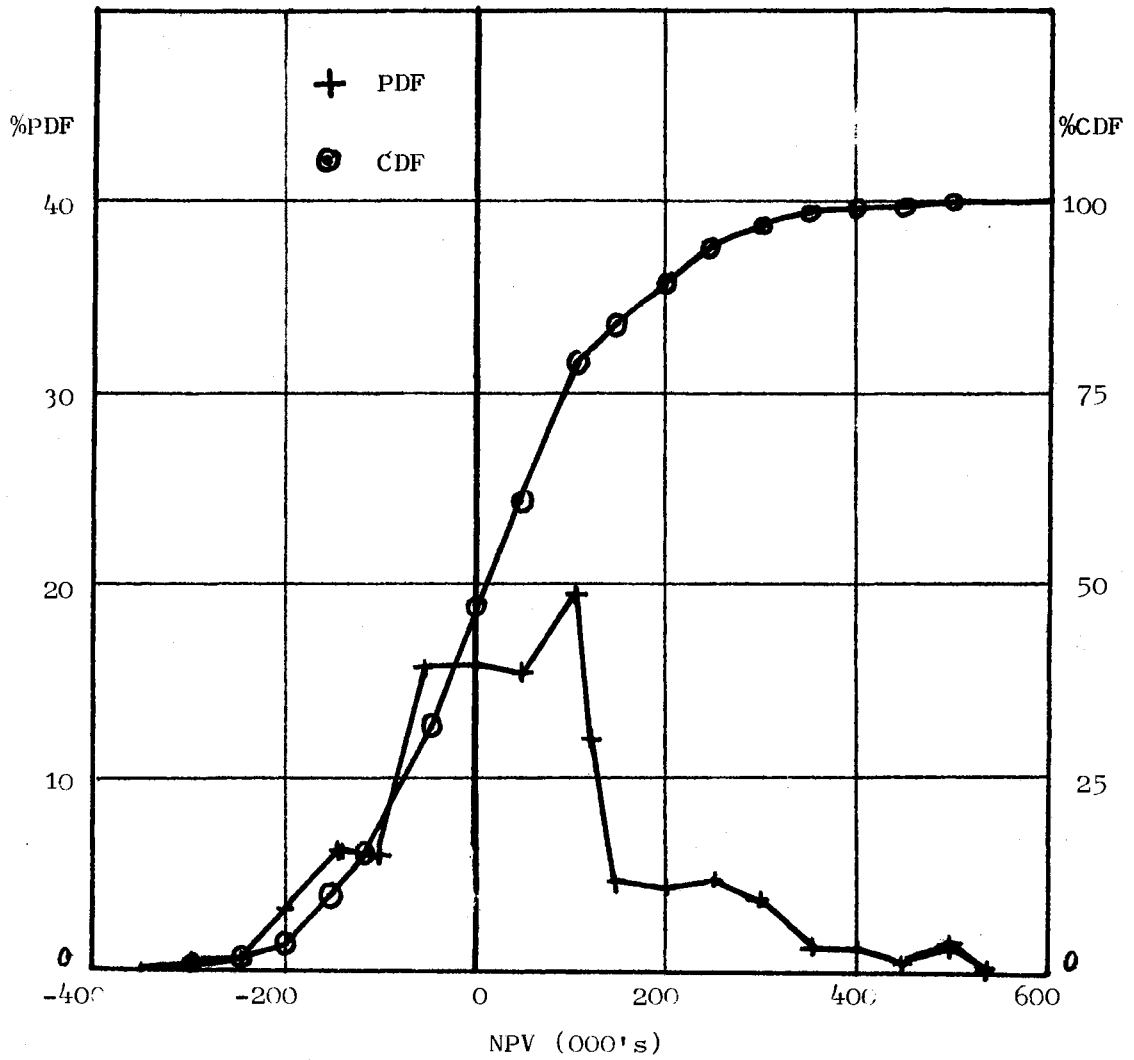


Figure 8. Probability Distribution of the NPV at 10% Discount Rate for the INCO Company Investment Project

(see Appendix A.1 for detailed information of how to run PRODIP). The results of running PRODIP indicate that Pearson distribution #3 (Gamma) is the best fit out of the Pearson system (Option A of PRODIP). Out of the standard statistical distributions allowed in Option B, the Beta distribution is found to be the best fit having the lowest χ^2 test and the K-S test.

5.7 Utility of the Investment

INCO measures the utility of its investment opportunities using the Bussey's model. INCO's parameters for the Bussey's utility model are given as follows:

$$U = a \cdot V_1 - b \cdot V_2 + cV_3 - dV_4$$

$$a = 48$$

$$b = 8$$

$$c = 2.75$$

$$d = 0.12$$

where (for the three models):

U = expected utility,

V = expected NPV $\times 10^{-5}$,

V_1 = 1st non-central moment of NPV $\times 10^{-5}$,

V_2 = 2nd non-central moment of NPV $\times 10^{-5}$,

V_3 = 3rd non-central moment of NPV $\times 10^{-5}$, and

V_4 = 4th non-central moment of NPV $\times 10^{-5}$.

PRODIP, in addition to fitting a distribution to the data, it also provides information about the central and non-central moments of the NPV distribution. From the PRODIP output, Appendix B, pp. 217-230, the following information is calculated from the four distribution moments

for Bussey's model:

$$V_1 = 0.50125,$$

$$V_2 = 2.53910,$$

$$V_3 = 4.63680, \text{ and}$$

$$V_4 = 21.63940.$$

The above information is fed into the program RUM to evaluate the different utilities. The input sheets for RUM are shown in Appendix B, pp. 231-232. The output results are given in Appendix B, p. 233. These utility results can be analysed for making the investment decisions using the rules in Sections 3.2.1 and 3.2.2.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

This research provides a basic model and a tool for analyzing investments under risk. The four computer modules of the UPFAR (Utility Program For the Analysis of Risk) system developed in this research can be used separately to provide a basic function needed for research other than risk analysis as well. The four modules that UPFAR offers are: 1) a Probability Distribution Fitting Program (PRODIP), 2) a General Utility Simulation System (GUSS), 3) an Investment Model Subroutine (INMO), and 4) a Routine for Utility Measurement (RUM).

The Probability Distribution Fitting Program (PRODIP) fits probability distributions to empirical stochastic data. PRODIP offers three main options. Option A of PRODIP generates the best distribution from the Pearson family that fits the data. Option B of PRODIP gives the best fit to the data to one or more of seven continuous distributions as requested. These distributions are the Exponential, Normal, Lognormal, Fisher-Tippett I, Fisher-Tippett II, Weibull, and the Beta. Option C of PRODIP gives the best fit to the data to one or more of five discrete distributions as requested. These distributions are the Poisson, Truncated Poisson, Binomial, Truncated Binomial, and the Truncated Negative Binomial. In addition to giving the first eight central and non-central moments of the data, PRODIP gives the parameters for each fitted distribution. Furthermore, PRODIP gives the values for

the χ^2 and the Kolmogorov-Smirnov (K-S) tests. These tests indicate the acceptability of the hypothesis that a fitted distribution represents the data. The χ^2 and the K-S tests can also be used as the basis for choosing the best fit to the data (the one with minimum χ^2 and/or K-S value) out of the tested distributions.

The General Utility Simulation System (GUSS) can be used as a general purpose simulator for analyzing different problems by linking it with a subroutine giving the problem's model relationships. GUSS uses the Monte Carlo simulation techniques. Of its main features are its flexibility and its being easy to use. To simulate a given problem, the analyst writes a subroutine representing a model of the problem under study. GUSS assumes the responsibility of receiving the input data, sampling from the different input distributions, housekeeping of the output results, and giving the requested reports and statistics. GUSS links with a problem subroutine through the use of variables with common names and defined in the common storage area.

The Investment Model (INMO) can be used to analyze a wide range of investment problems to study the risk of investment prospects. INMO includes the mathematical relationships necessary to calculate the cash flows from the different components of an investment. INMO also calculates the net present value of the investment for up to seven discounting rates. INMO has the capability of analyzing capital assets and calculating depreciation according to the Straight Line, Double Rate Declining Balance or the Sum of the Years Digits method of depreciation. INMO automatically replaces equipment after their service life with identical pieces until the end of the project's life. For the revenues and expenses, INMO allows a time lag before generating the revenue or

incurring the expense, and a growth period to their maturity levels. INMO calculates the net present value and passes information to GUSS for housekeeping.

The RUM module, a Routine for Utility Measurement, can be used to evaluate an investment's utility based on different utility models. Three utility models are included in RUM at the present time. These utility models are the Farrer's model, the Cramer and Smith's model, and the Bussey's model. RUM uses a utility model's coefficients and the investment's distribution parameters to calculate the utility of the investment.

A major value of the UPFAR system is in the area of analyzing investments under risk. Instead of looking at a single estimate of a net present value of an investment opportunity, UPFAR allows the analyst to look at a whole range of possible outcomes with their associated likelihood of occurrence. This would give the analyst further insights into the investment opportunity by explicitly introducing the uncertainties into the analyses. UPFAR can also be used for management training giving them a feeling and understanding of the concepts of risk analysis and how the uncertainty about some of the elements of the investment can affect their decisions.

The analyst should be aware that risk analysis would not eliminate uncertainty and would not guarantee that a specific decision is best. Rather, risk analysis recognizes the magnitude of uncertainty in the different variables together with their likelihoods. This would give a clearer understanding of the elements affecting an investment decision.

Extension to this research can be done in four areas. The first suggested area is to increase the degree of generality of the INMO

module to enable it to handle a wider range of investment problems. For example, INMO can be expanded to 1) allow other depreciation methods in addition to the three already available, 2) allow replacement of equipment that retire during the life of the project with non-identical ones, 3) allow a non-linear growth to maturity for revenues and expenses, and 4) allow a variable function for the maturity period.

A second area for research is to develop other INMO modules that can be applicable to special types of investment problems that the present INMO cannot handle. These INMO modules can be used to analyze different investment problems giving more insights into the investment process.

A third area for research is using PRODIP to identify and evaluate the probability distributions of common investment parameters as well as other parameters based on empirical data.

A fourth area which needs further research is the area of utility functions. Research can be directed toward evaluating the different parameters that can fit a given utility model and represent an investor's attitude, both on an individual and corporate basis.

REFERENCES

- (1) English, J. Morley. "A Discount Function for Comparing Economic Alternatives." Journal of Industrial Engineering, Vol. 16, No. 2 (March-April, 1965), pp. 115-118.
- (2) Solomon, E. "Measuring a Company's Cost of Capital." Journal of Business (October, 1955), pp. 240-252.
- (3) Hillier, Fredrick. "The Derivation of Probabilistic Information for the Evaluation of Risky Investments." Management Science, Vol. 9, No. 4 (April, 1963), pp. 443-457.
- (4) Hillier, Fredrick. "Supplement to the Derivations of Probabilistic Information for the Evaluation of Risky Investments." Management Science, Vol. 11, No. 3 (January, 1965), pp. 485-487.
- (5) Hillier, Fredrick, and David V. Helbink. "Evaluating Risky Capital Investments." California Management Review (Winter, 1965-66), pp. 71-80.
- (6) Canada, John R., and Harrison W. Wadsworth. "Methods for Quantifying Risk in Economic Analyses of Capital Projects." Journal of Industrial Engineering, Vol. 19, No. 1 (January, 1968), pp. 32-37.
- (7) Horowitz, Ira. "The Plant Investment Decision Revisited." Journal of Industrial Engineering, Vol. 17, No. 8 (August, 1966), pp. 416-422.
- (8) Hess, Sidney W., and Harry A. Quigley. "Analysis of Risk in Investments Using Monte Carlo Techniques." Chemical Engineering Progress Symposium Series 42: Statistics and Numerical Methods in Chemical Engineering, New York: American Institute of Chemical Engineers, 1963.
- (9) Hertz, David B. "Risk Analysis in Capital Investments." Harvard Business Review (Jan.-Feb., 1964), pp. 95-106.
- (10) Hertz, David B. "Investment Policies that Pay Off." Harvard Business Review (Jan.-Feb., 1968), pp. 96-108.
- (11) Bussey, Lynn E. "Capital Budgeting Project Analysis and Selection with Complex Utility Functions." (Unpub. Ph.D. thesis, Oklahoma State University, July, 1970).

- (12) Bussey, Lynn E., and G. T. Stevens, Jr. "Net Present Value from Complex Cash Flow Streams by Simulation." AIIE Transactions, Vol. 3, No. 1 (March, 1971), pp. 81-87.
- (13) Reisman, Arnold, and A. K. Rao. Discounted Cash Flow Analysis: Stochastic Extension. Monograph Series # 1, Engineering Economy Division, American Institute of Industrial Engineers, Inc., Atlanta Norcross, Georgia, 1973.
- (14) Hicks, John R., and R. G. D. Allen. "A Reconsideration of the Theory of Value." Economica, New Series I (Feb., 1934), pp. 52-76; and Ibid, I (May, 1934), pp. 196-219.
- (15) Marshall, Alfred. Principles of Economics. London: MacMillan and Company, Ltd., 1920.
- (16) Von Neumann, John, and Oskar Morgenstern. Theory of Games and Economic Behavior, 2nd ed. Princeton, N. J.: Princeton University Press, 1947.
- (17) Markowitz, Harry M. Portfolio Selection: Efficient Diversification of Investments. New York, N. Y.: John Wiley and Sons, Inc., 1959.
- (18) Farrer, Donald E. The Investment Decision Under Uncertainty. Englewood Cliffs, N. J.: Prentice Hall, Inc., 1962.
- (19) Freund, Rudolf J. "The Introduction of Risk Into a Programming Model." Econometrica, Vol. 24, No. 3 (July, 1956), pp. 253-263.
- (20) Cramer, Robert H., and Bernard E. Smith. "Decision Models for the Selection of Research Projects." The Engineering Economist, Vol. 9, No. 2 (Winter, 1964), pp. 1-20.
- (21) Swalm, Ralph O. "Utility Theory--Insights Into Risk Taking." Harvard Business Review (Nov.-Dec., 1966), pp. 123-136.
- (22) Raiffa, Howard. "Risk Ambiguity and the Savage Axioms: Comment." Quarterly Journal of Economics (Nov., 1961), pp. 690-694.
- (23) Raiffa, Howard. Decision Analysis. Reading, Mass.: Addison-Wesley Publishing Co., 1968.
- (24) Falls, Lew W. A Computer Program for Standard Statistical Distributions. NASA Technical Memorandum #TMX-64588 (April 30, 1971).
- (25) Hahn, Gerald J., and Samuel S. Shapiro. Statistical Models in Engineering. New York, N.Y.: John Wiley and Sons, Inc., 1967.
- (26) Merrington, M., and E. S. Pearson. "An Approximation to the Distribution of Non-Central t." Biometrika, No. 45 (1958), p. 484.
- (27) Lindgren, B. W. Statistical Theory. New York: The MacMillan Company, 1968.

- (28) Chernoff, H., and L. Moses. Elementary Decision Theory.
New York: John Wiley and Sons, 1959.

A SELECTED BIBLIOGRAPHY

- Alchian, Armen A. "The Meaning of Utility Measurement." American Economic Review, Vol. 7, No. 3 (March, 1953), pp. 26-50.
- Baxter, W. T., and N. H. Carrier. "Depreciation, Replacement Price, and Cost of Capital." Journal of Accounting Research, Vol. 9, No. 2 (Autumn, 1971), pp. 189-214.
- Berger, R. W. "Use Risk Analysis for Decision Making." Computer Decisions, Vol. 4, No. 3 (March, 1972), pp. 18-22.
- Bergthaller, C. "A Quadratic Equivalent of the Minimum Risk Problem." Review Roumaine de Mathematique Pures et Appliquees, Vol. 15, No. 1 (1970), pp. 17-23.
- Bierman, Harold, Jr., Charles P. Bonini, Lawrence E. Fouraker, and Robert K. Jaedicke. Quantitative Analysis for Business Decisions, rev. ed. Homewood, Ill: Richard D. Irwin, Inc., 1965.
- Borch, Karl. "A Note on Utility and Attitudes to Risk." Management Science, Vol. 9, No. 4 (July, 1963), pp. 697-700.
- Bussey, Lynn E. "Simulation of Complex Cash Flow Streams in Capital Budgeting Project Analysis." (Unpublished report, School of Industrial Engineering and Management, Oklahoma State University, 1970.)
- Byrne, R., A. Charnes, W. W. Cooper, and K. Kortanek. "A Chance-Constrained Approach to Capital Budgeting with Portfolio-Type Payback and Liquidity Constraints and Horizon Posture Controls." Journal of Financial and Quantitative Analysis, Vol. 2 (December, 1967), pp. 339-364.
- Canada, John R. "Decision and Risk Analysis--A Review and Critique." Proceedings of AIIE 25th Annual Meeting (June, 1973), pp. 37-40.
- Charnes, A., W. W. Cooper, and M. H. Miller. "Application of Linear Programming to Financial Budgeting and Costing of Funds." Journal of Business, Vol. 32, No. 1 (January, 1959), pp. 20-46.
- Daniel, R. C. "Phasing Out Capital Equipment." Operations Research Quarterly, Vol. 24, No. 1 (March, 1973), pp. 113-116.
- Dean, Joel. Capital Budgeting. New York: Columbia University Press, 1951.

- DeFinetti, Bruno. "Foresight: Its Logic Laws, Its Subjective Sources." Studies in Subjective Probability, ed. H. E. Klyberg and H. E. Smokler. New York: John Wiley and Sons, Inc., 1964.
- Eilon, S., and T. R. Fowkes. "Sampling Procedure for Risk Simulation." Operations Research Quarterly, Vol. 24, No. 2 (June, 1973), pp. 241-253.
- Firedman, M., and L. J. Savage. "The Utility Analysis of Choices Involving Risk." Journal of Political Economy (April, 1948), pp. 279-304.
- Fishburn, Peter C. Decision and Value Theory. New York: John Wiley and Sons, Inc., 1964.
- Fishman, G. S. "Estimating Sample Size in Computing Simulation Experiments." Management Science, Vol. 18, No. 1 (September, 1971), pp. 21-38.
- Fleischer, Gerald A. "Major Issues Associated with the Rate of Return Method for Capital Allocation: The 'Ranking Error' and 'Preliminary Selection.'" Journal of Industrial Engineering, Vol. 17, No. 4 (April, 1966), pp. 202-208.
- Freund, Rudolf J. "The Introduction of Risk Into a Program Model." Econometrica, Vol. 24, No. 3 (July, 1956), pp. 253-263.
- Grayson, C. Jackson, Jr. Decisions Under Uncertainty: Drilling Decisions by Oil and Gas Operators. Boston: Harvard University, Division of Research, Graduate School of Business Administration, 1960.
- Green, Paul E. "Risk Attitudes and Chemical Investments Decisions." Chemical Engineering Progress, Vol. 59, No. 1 (January, 1963), pp. 35-40.
- Harvey, Roger Kent. "A Portfolio Model of Capital Budgeting Under Risk." (Unpublished Ph.D. dissertation, School of Business, Indiana University, 1967.)
- Hertz, David B. "Risk Analysis in Capital Investment." Harvard Business Review (January-February, 1964), pp. 95-106.
- Hertz, David B. "Investment Policies That Pay Off." Harvard Business Review (January-February, 1968), pp. 96-108.
- Johnson, N. L., E. Nixon, and D. E. Amos. "Table of Percentage Points of Pearson Curves for Given β_1, β_2 , Expressed in Standard Measure." Biometrika, Vol. 50 (1963), pp. 459-498.
- Kobak, I. W., and J. Owen. "Random Variables, The Time Value of Money and Capital Expenditures." Management Science, Vol. 17, No. 3 (Nov., 1970), pp. 142-145.

- Leibson, I., and C. Trischman, Jr. "Decision Trees: A Rapid Evaluation of Investment Risk." Chemical Engineering, Vol. 79, No. 2 (Jan., 1972), pp. 99-106.
- Leung, T. K. Y. "New Monograph: Quick Route to Discounted Cash Flow." Chemical Engineering, Vol. 77, No. 12 (June, 1970), pp. 208-209.
- Lietaer, Bernard A. "Managing Risks in Foreign Exchange." Harvard Business Review (March-April, 1970), pp. 127-138.
- Lorie, James H., and Leonard J. Savage. "Three Problems in Rationing Capital." Journal of Business, Vol. 28, No. 4 (October, 1955), pp. 229-239.
- Luce, R. D., and H. Raiffa. Games and Decisions. New York: John Wiley and Sons, Inc., 1957.
- Lusztig, P., and B. Schwab. "A Note on the Application of Linear Programming to Capital Budgeting." Journal of Financial and Quantitative Analysis, Vol. 3, No. 4 (Dec., 1968), pp. 427-431.
- Mao, James C. T. Quantitative Analysis of Financial Decisions. Toronto: The Macmillan Company, Collier-Macmillan Canada, Ltd., 1969.
- Masse, Pierre. Optimal Investment Decisions. N. J.: Prentice-Hall, Inc., 1962.
- McLean, John G. "How to Evaluate New Capital Investments." Harvard Business Review (Nov.-Dec., 1958), pp. 59-69.
- Meyer, R. A., Jr. "Equipment Replacement Under Uncertainty." Management Science, Vol. 17, No. 11 (July, 1971), pp. 750-758.
- Mihram, G. A. "Some Practical Aspects of the Verification and Validation of Simulation Models." Operations Research Quarterly, Vol. 23, No. 1 (April, 1972), pp. 17-29.
- Moder, J. J., C. Gables, and J. J. Nickl. "An Optimal Allocation of Resources for the Capital Budgeting Process." Socio-Economic Planning Science, Vol. 3 (1969), pp. 135-150.
- Mosteller, Frederick, and Philip Noguee. "An Experimental Measurement of Utility." The Journal of Political Economy, Vol. 59, No. 5 (October, 1951), pp. 371-404.
- Mulder, K. J. "Discounted Cash Flow Analysis and Investment Policy." Management International Review, Vol. 9, No. 4/5 (1969), pp. 131-141.
- Pearson, A. W. "A Note on Project Selection in Research and Development --Post-Rothchild." Operations Research Quarterly, Vol. 24, No. 2 (June, 1973), pp. 295-298.

- Ramsey, Frank P. "Truth and Probability" (1926), in Studies in Subjective Probability, ed. H. E. Klyberg and H. E. Smokler. New York: John Wiley and Sons, Inc., 1964.
- Reisman, Arnold, and Arza K. Rao. Discounted Cash Flow Analysis: Stochastic Extensions. American Institute of Industrial Engineers, Inc., Norcross, Georgia: 1973.
- Ross, R. C. "Uncertainty Analysis Helps in Making Business Decisions." Chemical Engineering, Vol. 78, No. 21 (Sept., 1971), pp. 149-155.
- Russel, A. H. "Cash Flows In Networks." Management Science, Vol. 16, No. 5 (Jan., 1970), pp. 357-373.
- Schlaifer, Robert O. Analysis of Decisions Under Uncertainty. New York: McGraw-Hill, 1967.
- Shuford, E., Lyle V. Jones, and R. Darrell Bock. "A Ratio Scale for Utility." (Unpublished paper presented at the 1959 meeting of the American Psychological Association.)
- Spetzler, Carl S. "Establishing a Corporate Risk Policy." Pre-print of paper presented at the 19th Annual Conference of the American Institute of Industrial Engineers, Summer, 1968.
- Swalm, Ralph O. "Utility Theory--Insights Into Risk Taking." Harvard Business Review (November-December, 1966), pp. 123-136.
- Thorneycroft, W. T., J. W. Greener, and H. A. Patrick. "Investment Decisions Under Uncertainty and Variability--Some Practical Experiences of Using Forecasts and Probabilities." Operations Research Quarterly, Vol. 19, No. 2 (June, 1968), pp. 143-160.
- Thorngren, J. T. "Probability Technique Improves Investment Analysis." Chemical Engineering, Vol. 74, No. 17 (Aug. 14, 1967), pp. 143-151.
- Tribus, Myron, Robert Evans, and Gary Crellin. "The Use of Entropy In Hypothesis Testing." The Tenth National Symposium on Reliability and Quality Control, January 7-9, 1964.
- Unger, V. E., Jr. "Capital Budgeting and Mixed Zero-One Integer Programming." AIIE Transactions, Vol. 2, No. 1 (March, 1970), pp. 28-38.
- Watters, Lawrence J. "Research and Development Project Selection: Inter-Dependence and Multi-Period Probabilistic Budget Constraints." (Unpublished Ph.D. dissertation, Arizona State University, 1967.)
- Weiss, Dieter. "Investment Planning for an Electric Energy System at Tunesia With a Simulation Model (in French)." Review Francaise d'Informatique et de Research Operationnelle, Vol. 4, No. 2 (June, 1970), pp. 97-110.

Whitmore, G. A., and S. Darkazanli. "A Linear Risk Constraint in Capital Budgeting." Management Science, Vol. 18, No. 4, Part I (Dec., 1971), pp. B-155-157.

Winegartner, H. Martin. Mathematical Programming and the Analysis of Capital Budgeting Problems. Original edition. Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1963.

Winegartner, H. Martin. Mathematical Programming and the Analysis of Capital Budgeting Problems. Markham edition. Chicago: Markham Publishing Co., 1967.

Winegartner, H. Martin. "Capital Budgeting of Interrelated Projects: Survey and Synthesis." Management Science, Vol. 12, No. 7 (1966), pp. 485-516.

APPENDIX A

UPFAR USER'S GUIDE

The use of the UPFAR system is a simple task as a result of its modular design, flexibility, and ease of its input/output procedures. The major attention in a risk analysis or other simulation problems should be directed to the conceptual aspects of the problem under study while preparing the input, choosing certain options of the model, and to the analysis of the output. Verification that a chosen distribution, the INMO model, and the chosen utility model, do represent the real life problem is essential. Otherwise, the output results, regardless of how good they look, become of no value.

To use the UPFAR modules, the following procedure is recommended:

- 1) Identify the different variables and parameters that are pertinent to the problem.
- 2) Ascertain that the input data are representative of the variable under study and that no extraneous observations or extreme cases are included.
- 3) Identify the characteristics of the variable under study and ascertain that the variable is not being misrepresented by a chosen distribution.
- 4) Validate that the resultant distribution that best fits the data agrees with real life situations.
- 5) Ensure that you are using the appropriate INMO and that your problem is being represented by the model.
- 6) Choose the number of simulation iterations that would provide the desired accuracy.
- 7) Ascertain that the chosen utility model and its parameters represent the investor's attitudes.

- 8) When presenting the results of the study, make the decision maker aware that:
- a) There is no assurance that the outcome of a recommended decision is guaranteed.
 - b) Risk analysis does not eliminate uncertainty. It rather leads to a clearer understanding of the result of the interaction between the decision elements.

Figure 9 gives the flow chart of UPFAR.

A.1 PRODIP User's Guide

PRODIP offers efficient procedures to fit statistical distributions to empirical data. The program calculates the central and non-central moments, appropriate variances and standard errors of theoretical parameters, and the cumulative probability function.

PRODIP offers three alternative options. Option A gives the best distribution out of the Pearson System of frequency distributions. Option B provides seven alternative standard continuous distributions. Option C provides five alternative standard discrete distributions. PRODIP also provides statistical tests for "goodness of fit" for making objective decisions about the acceptance of the fitted distribution. Figure 10 gives the flow chart of PRODIP.

A.1.1 Input Data Preparation

The input can be classified into three main parts:

- 1) Request identification title.
- 2) Required options A, B, and/or C.
- 3) Frequency data.

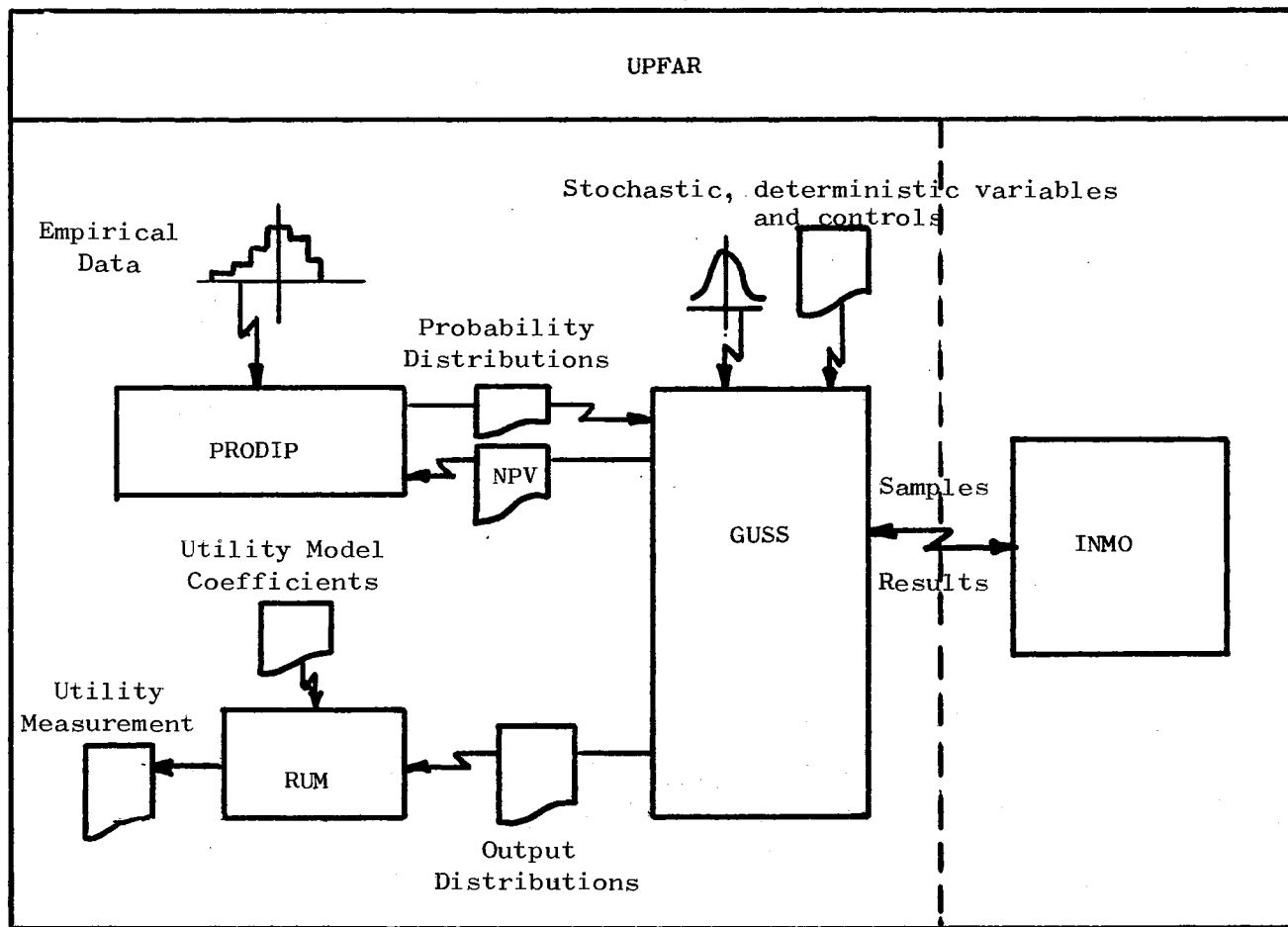


Figure 9. Utility Program For Analysis of Risk (UPFAR)--Flow Chart

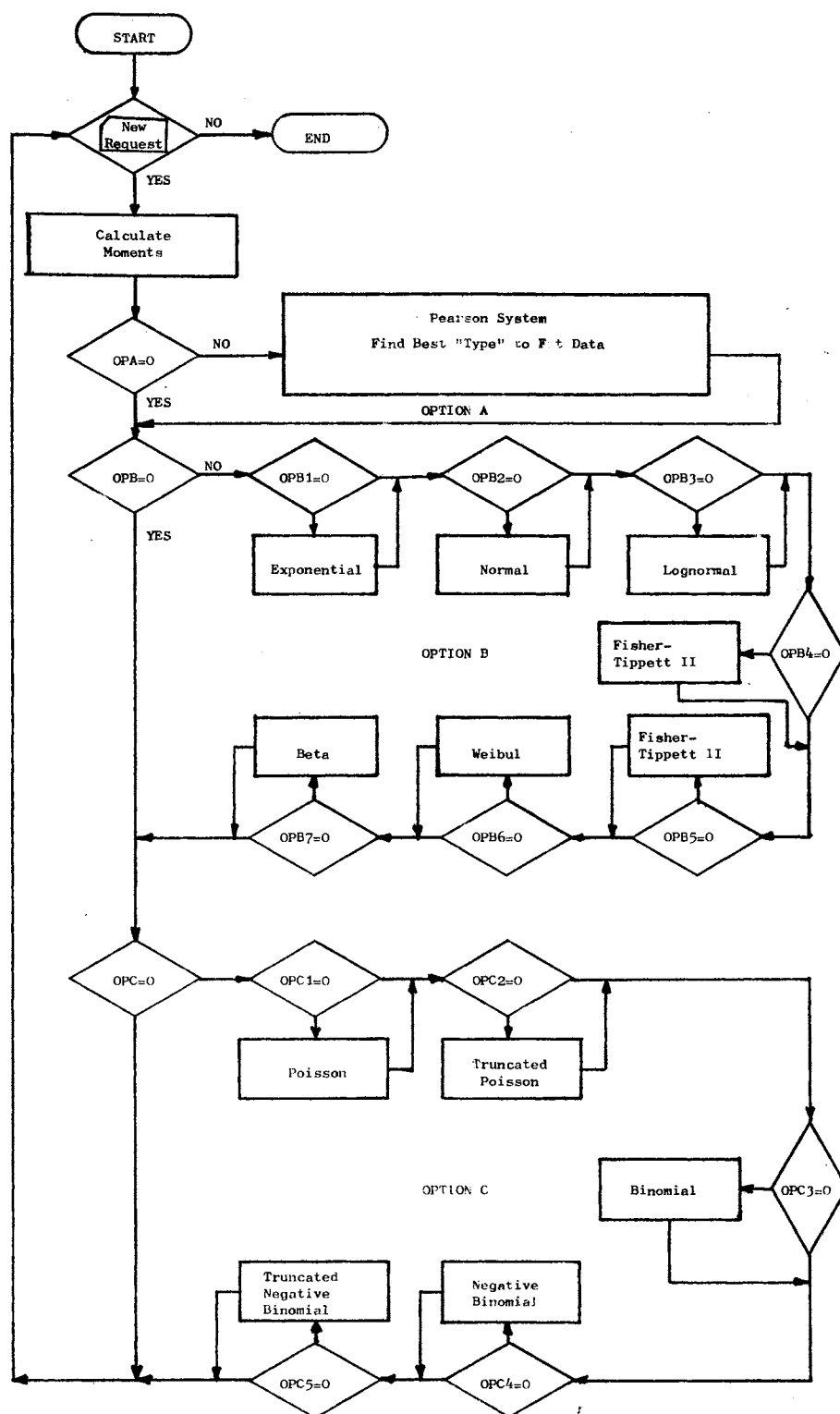


Figure 10. Probability Distribution Fitting Program
(PRODIP) -- Flow Chart

Appendix B, pp. 122-124, gives the input sheets for PRODIP.

- a) Card # 1: Request Identification Title:

FORMAT (20A4)

This is simply a title for the run identification. The full length of the card (80 characters) is allowed for the run ID. The title should be centered about the middle of the card (column # 40) for better appearance on the program output.

- b) Card # 2: Option A

FORMAT (9X,I1)

Enter "1" in column 1 of this card if Option A, the Pearson distributions, is desired and "0" otherwise. PRODIP selects the best fit out of the Pearson System of distributions.

- c) Card # 3: Option B

FORMAT (8(9X,I1))

Enter "1" in column 1 of this card if any of the continuous distributions is desired, "0" otherwise. If "1" is entered in column 10, indicate which of the continuous distributions is desired by entering "1" in the appropriate column as indicated below, and "0" if not requested:

<u>Column #</u>	<u>Distribution</u>
20	Exponential
30	Normal
40	Lognormal
50	Fisher-Tippett I
60	Fisher-Tippett II
70	Weibull
80	Beta

d) Card # 4: Option C

FORMAT (8(9X,I1))

Enter "1" in column 10 of this card if any of the discrete distributions is desired, "0" otherwise. If "1" is entered in column 10, indicate which of the discrete distributions is desired by entering "1" in the appropriate column as indicated below, and "0" if not requested:

<u>Column #</u>	<u>Distribution</u>
20	Poisson
30	Truncated Poisson
40	Binomial
50	Negative Binomial
60	Truncated Negative Binomial

e) Card # 5: Data Controls

FORMAT (2I10,4E10.0,I10)

This card has the different controls about the data as follows:

<u>Field Columns</u>	<u>Description</u>
1-10	N = Total number of observations (sample size)
11-20	NV = Number of observation groups
21-30	H = Length of observations class intervals for Sheppard's correction when calculating the relative measures of skewness and Kurtosis, '0' if not needed.
31-40	SMFE = Smallest frequency for Chi-Square test. SMFE can be assumed = 5.0 unless user specifies otherwise.
41-50	HY = Length of class interval used for the lognormal distribution. If HY = 0.0, the sample standard deviation is computed the normal way. If HY > 0.0, otherwise it will be computed using Sheppard's correction on M2.

51-60

P = used with the binomial distribution. When binomial is requested, P must be greater than zero.

61-70

LR = Highest outcome for the binomial distribution if the binomial is requested.

f) Cards # 6, 7, etc: Sample Information:

FORMAT (4(15E2/))

Cards from the 6th up, until the end of data for this run, will include the information about each observation class or group, each point on a separate card in an ascending order by XB and XT, as follows:

<u>Field Columns</u>	<u>Description</u>
1-15	XB = Bottom value of class interval
16-30	XT = Top value of class interval
31-45	XI = Class mark, or value of observation
46-60	FI = Frequency of the observation

g) Multiple Runs:

The program allows multiple runs for fitting distributions to different parameters of interest. Simply add additional decks of data, in the same order as before, for the other parameters of interest.

A.1.2 PRODIP Sample Run

Appendix B, pp. 125-127, shows the input for a sample run using PRODIP to check all its available options. Appendix B, pp. 128-146, shows the output giving the different distributions.

The example given here is merely an illustration of using PRODIP. Since no significance is placed on the nature of the appropriate type of distribution to fit the data, a request to run all options is used here. The reader is reminded that a particular distribution, or a class of distributions, e.g., continuous or discrete, should be requested depending on the nature of the variable that the data represent.

A.1.3 Remarks on Using PRODIP

The user of PRODIP should be aware that it is only a tool to fit probability distributions to the data. It is the responsibility of the user then to make the decisions of whether or not certain distributions should or should not be considered. For examples:

- 1) If the variable under consideration is discrete in nature, it would be inappropriate to run options A and B. On the other hand if the variable is continuous, options A and B are the ones to be tried.
- 2) If the user is grouping the data into classes rather than using their actual values, the use of Sheppard's correction factor [27, 28] may be appropriate.
- 3) When choosing among the different eligible distributions for a best fit, the appropriate test of goodness fit should be used. Two tests are given here, the χ^2 test and the Kolmogorov-Smirnov (K-S) test which uses the maximum absolute difference (MAD).
- 4) When using the χ^2 test, the user should be aware that theoretical aspects of the χ^2 distribution require that the

number of classes be ≥ 5 and that each expected class frequency be ≥ 5 . Therefore, when an expected class frequency is < 5 , then expected frequencies of adjacent classes should be combined together. Selection between the different distributions can be decided according to which distribution fit gives the minimum χ^2 value. The χ^2 value is also tested against the χ^2 distribution index for the given number of degrees of freedom in order to accept or reject the hypothesis that the distribution provides a good fit for the data.

- 5) The K-S test is restricted to continuous distributions. However, in cases where the χ^2 test cannot be used because of small number of classes or negative degrees of freedom for example, the K-S test may be used as a guide.

When comparing different fits on the K-S test, the best fit is the one with minimum MAD.

- 6) The user is referred to Hahn and Shapiro [25], or other references regarding the choice of the form or type of distribution to use in a given situation.

A.1.4 PRODIP Dictionary

Following is list of variables and their meanings as used in the input to, and output of, PRODIP.

- a) Input:

PID = Problem Identification Title.

OPA = Selector for option A; 0 \equiv skip, 1 \equiv desired.

OPB = Selector for option B; 0 \equiv skip, 1 \equiv any of the continuous distributions is desired.

- OPB1 = Selector for Exponential distribution; 0 \equiv skip,
1 \equiv desired.
- OPB2 = Selector for Normal distribution; 0 \equiv skip, 1 \equiv desired.
- OPB3 = Selector for Lognormal distribution; 0 \equiv skip, 1 \equiv desired.
- OPB4 = Selector for Fisher-Tippett I distribution; 0 \equiv skip,
1 \equiv desired.
- OPB5 = Selector for Fisher-Tippett II distribution; 0 \equiv skip,
1 \equiv desired.
- OPB6 = Selector for Weibull distribution; 0 \equiv skip, 1 \equiv desired.
- OPB7 = Selector for Beta distribution; 0 \equiv skip, 1 \equiv desired.
- OPC = Selector for option C; 0 \equiv skip, 1 \equiv any of the discrete distributions is desired.
- OPC1 = Selector for Poisson distribution; 0 \equiv skip, 1 \equiv desired.
- OPC2 = Selector for Truncated Poisson distribution; 0 \equiv skip,
1 \equiv desired.
- OPC3 = Selector for Binomial distribution; 0 \equiv skip, 1 \equiv desired.
- OPC4 = Selector for Negative Binomial distribution; 0 \equiv skip,
1 \equiv desired.
- OPC5 = Selector for Truncated Negative Binomial; 0 \equiv skip,
1 \equiv desired.
- N = Total number of observations in the sample.
- NV = Number of observation groups.
- H = Length of observations class intervals used for
Sheppard's correction when calculating B1 and B2; the
relative measures of skewness and Kurtosis (see nomen-
clature). If H is zero, B1 and B2 will be computed using
uncorrected moments.

SMFE = Smallest frequency for Chi-Square test. SMFE can be assumed = 5.0 in the program unless user desires otherwise.

HY = Length of class interval, used for the Lognormal distribution. If HY = 0.0, S is computed the normal way. If HY > 0.0, S will be computed using Sheppard's correction on M2 (see nomenclature).

P = Used with the Binomial distribution. When Binomial is requested, P > 0 and all the observations must be integers.

LR = Highest outcome for the Binomial distribution when requested.

XI = Value of observation, class mark.

FI = Frequency of a given XI.

XB = Bottom of class interval.

XT = Top of class interval.

b) Output:

PM1 ... PM8 = 1st ... 8th non-central moments of the sample

$$(m'_1 \dots m'_8).$$

M1 ... M8 = 1st ... 8th sample's central moments

$$(m_1 \dots m_8).$$

S = S_x = Sample's standard deviation.

BE1 = M_3^2/M_2^3 ($\sqrt{\text{BE1}}$ measures skewness relative to degree of spread).

BE2 = M_4/M_2^2 (BE2 = relative measure of Kurtosis).

$$\text{BE3} = (M_3 \cdot M_5)/M_2^4$$

$$BE4 = M_6/M_2^3$$

$$BE6 = M_8/M_2^4$$

$$K = \beta_1(\beta_2+3)^2/[4(\beta_2-3\beta_1)(2\beta_2-3\beta_1-6)]$$

$$B1 = BE1 = \frac{M_3^2}{M_2^3} (\sqrt{B1} \text{ measures skewness relative to degree of spread}).$$

$$B2 = BE2 = \frac{M_4}{M_2^2} (B2 \text{ relative measure of Kurtosis}).$$

$$\begin{aligned} \text{LAMBDA} = \lambda = \{ & (4\beta_2-3\beta_1)(10\beta_2-12\beta_1-18)^2 - \beta_1(\beta_2+3)^2(8\beta_2-9\beta_1-12) \} \\ & / \{ (3\beta_1-2\beta_2+6)[(\beta_1(\beta_2+3)^2 + 4(4\beta_2-3\beta_1)(3\beta_1-2\beta_2+6))] \} \end{aligned}$$

$$\text{SIGB1} = \sigma_{\beta_1}$$

$$\text{SIGB2} = \sigma_{\beta_2}$$

$$\text{SIGL} = \sigma_{\lambda}$$

$$\text{SIGK} = \sigma_K$$

$$A = 4\beta_2 - 3\beta_1$$

$$B = 10\beta_2 - 12\beta_1 - 18$$

$$C = \beta_2 + 3$$

$$D = 8\beta_2 - 9\beta_1 - 12$$

$$E = 3\beta_1 - 2\beta_2 + 6$$

$$F = 2\beta_2 - 3\beta_1 - 6$$

$$C1 = 5\beta_2 - 6\beta_1 - 9$$

$$C2 = 2\beta_2 - 3\beta_1 - 6$$

$$\text{SGC1} = \sigma_{C_1}$$

$$\text{SGC2} = \sigma_{C_2}$$

$$\text{CFI} = \text{Cumulative frequency.}$$

$$\text{DFO} = \text{Observed distribution function.}$$

$$\text{H(X)} = \text{Expected distribution function.}$$

FE = Expected frequency.

MAX ABS DIFF = Maximum absolute difference.

CHISQ = Sample χ^2 index.

NG(CHISQ) = Number of classes after grouping for χ^2 test.

DF = Number of degrees of freedom for χ^2 test.

A.2 GUSS User's Guide

GUSS is a general utility simulation system that uses Monte Carlo techniques. The program provides the user with an easy way to apply simulation for the analysis of problems.

In order to simulate a given problem, the analyst writes a subroutine representing the model of the problem. This model would include all the equations representing all the relationships between the different variables and their interactions. GUSS assumes the responsibility of receiving the input data for all the constants, deterministic variables, and the distributions of the stochastic variables. GUSS also receives all the controls regarding the housekeeping and specifications of the output reports. After receiving the input data, GUSS starts the simulation iterations and recycles from one iteration to the other until all the number of requested iterations, which is an input to GUSS, have been completed. In each iteration, GUSS samples from the different distributions of the stochastic variables, connects with the problem's model in the subroutine, receives the results of the output parameters, and tabulates the results of the simulation. After all the simulation iterations have been completed, GUSS sorts the results, calculates the distribution statistics for each output parameter, and presents the requested output reports. Figure 11 gives a flow chart of GUSS.

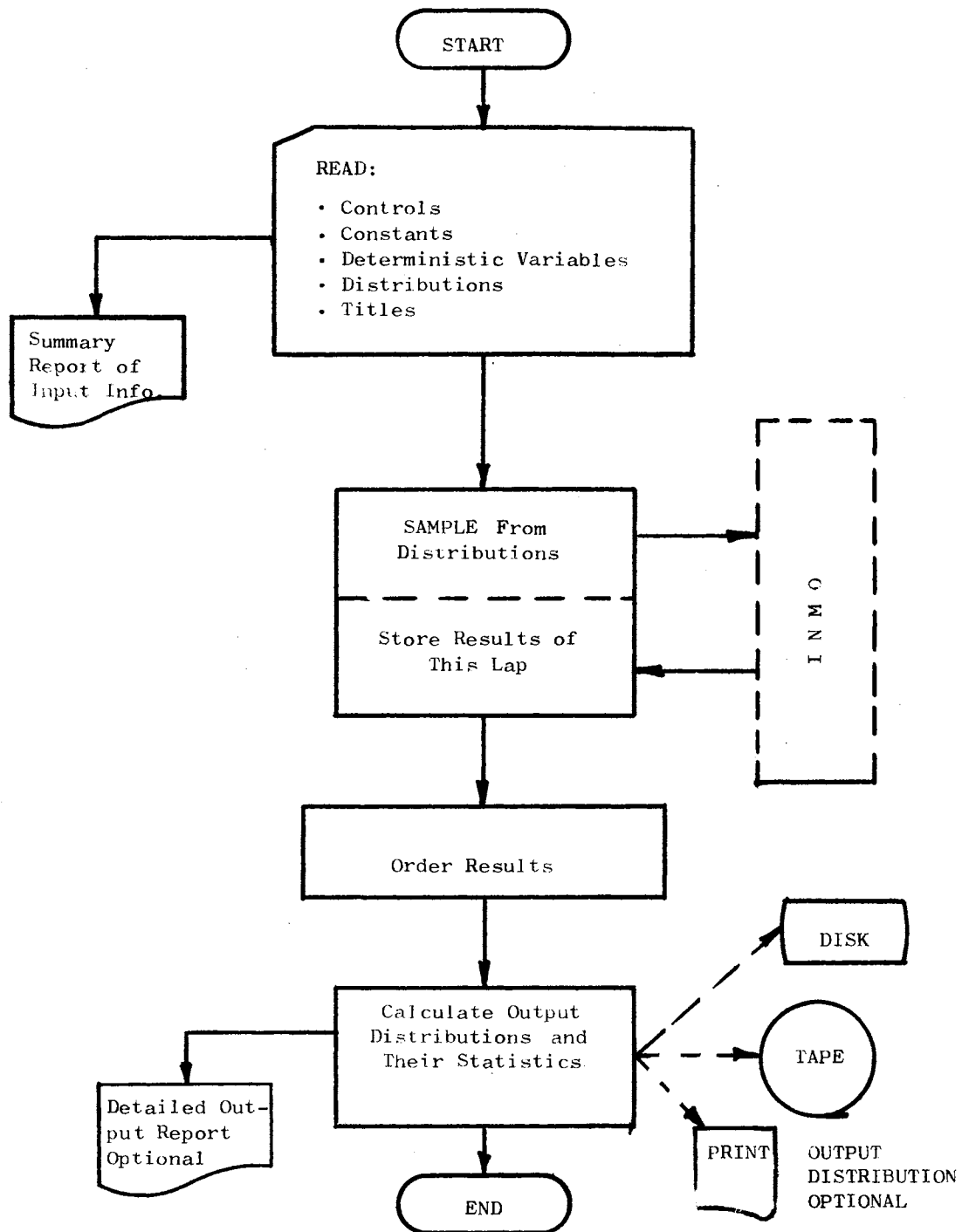


Figure 11. General Utility Simulation System (GUSS) --
Flow Chart

A.2.1 GUSS's Capabilities

GUSS has the capability of handling simulation problems with the following characteristics:

- Up to 50 integer constants.
- Up to 50 deterministic variables.
- Up to 50 stochastic variables; continuous or discrete.
- Up to 25 testing and acceptability criteria.
- Up to 1000 simulation iterations.
- Up to 7 output parameters.
- Up to 5 output devices for the output parameters' detailed distributions which can be suppressed from the printouts.
- Up to 10 counters that can be used during the simulation.
- Up to 10 reports that can be tailored by sorting and/or suppressing part of the simulation results according to the user's needs.
- Up to 10 different block sorts allowed with each report.
- Up to 40 footnotes can be stored in GUSS, of which a selected subset, or the whole set, can be printed as requested with the different reports. Twenty values can be calculated and printed opposite to 20 footnotes.
- A summary of each report including the 10% distribution values of the different output parameters together with its mean and variance.

One of GUSS's main features is its ease and flexibility. This is demonstrated in the data input stage, linkage with the problem's model subroutine, the report tailoring capabilities, and the organized report format. The simulation results are reported in the form of a final matrix tableau (output variables x simulation iterations). This tableau can be manipulated in too many different ways, as can be meaningful to

the problem at hand, by sorting the output results and tailoring the desired report. GUSS allows up to 10 tailored output reports. Each report can have up to 10 different sorts that may be necessary for that report.

A.2.2 Input Data Preparation

The input can be classified into the following main parts:

- 1) Problem identification title.
- 2) Input and output controls.
- 3) Problem constants.
- 4) Deterministic variables.
- 5) Stochastic variables.
- 6) Acceptability criteria.
- 7) Output controls.

Appendix B, pp. 147-152, gives the input sheets for GUSS.

- a) Card # 1: Problem Identification Title:

FORMAT (20A4)

This simply is a title for the run identification. The title is written on the full 80 columns of this card and should be centered about column # 40 for better appearance on the program output.

- b) Card # 2: Input/Output and Simulation Controls:

FORMAT (10I5)

This card includes the count number of input/output variables, simulation, and controls for results' manipulation. The data are structured as follows:

<u>Column #</u>	<u>Control Description</u>
2-5	number of simulation iterations -- maximum 1000.
9-10	number of integer constants used in the problem model -- minimum 0, maximum 50.
14-15	number of deterministic (floating point) variables (and/or constants) used in the problem model -- minimum 0, maximum 50.
19-20	number of stochastic variables used in the problem model -- minimum 1, maximum 50.
24-25	number of acceptability criteria used in model -- minimum 0, maximum 25.
29-30	number of output parameters requested by problem model -- minimum 1, maximum 7.
34-35	number of printout reports requested -- minimum 1, maximum 10.
39-40	number of footnotes to be printed below the output reports -- minimum 0, maximum 40.

c) Card # 3: Output Devices:

FORMAT (6I5)

Five optional output devices are allowed for the detailed simulation output distributions. These devices can be the high speed printer, disk files, magnetic tapes and/or the card punch. The high speed printer is given the code # 6 and the card punch is given code # 7. The code key for other devices, except for the numbers 5, 6, and 7 (card reader, high speed printer, and card punch), is arbitrary and is specified by the user in the JCL cards. The following are examples of JCL for a disk file, a magnetic tape, and the card punch, respectively.

DISK FILE:

```
//GO.FT01FOO1 DD UNIT=2314,VOL=SER=DISK05,
// DSN=OSU.ACTnnnnn.name,DISP=(NEW,KEEP),
// DCB=(LRECL=36,BLKSIW=1092,RECFM=VBC),
// SPACE=(40,1000)
```

MAGNETIC TAPE:

```
//GO.FT03FOO1 DD UNIT=TAPE,VOL=SER=Tnnnn,
// DSN=OSU.SCTnnnnn.name,DISP=(NEW,KEEP),
// DCB=(LRECL=36,BLKSIZE=1092,RECFM=VBS),
// LABEL=(1,SL)
```

CARD PUNCH (CODE #7):

```
//GO.SYSPUNCH DD SYSOUT=B
```

The following units are available now by GUSS according to the following code:

<u>Unit Code</u>	<u>Unit Specification</u>
1	2314 disk file
2	optional disk file
3	magnetic tape 7 track 800 BPI
4	magnetic tape 9 track 800 BPI
6	High speed printer, 132 columns
7	card punch

A summary of the output is always printed on the high speed printer irrespective of whether or not the detailed output is requested.

To specify the requested output devices, card # 3 should be filled as follows:

<u>Column #</u>	<u>Description</u>
5	number of output devices for detailed output, maximum 5. If 0 is entered, the detailed distributions will not be given and only the summary output will be given on the high speed printer.
10	Optional -- key code for 1st output device.
15	Optional -- key code for 2nd output device.
20	Optional -- key code for 3rd output device.
25	Optional -- key code for 4th output device.
30	Optional -- key code for 5th output device.

N.B. The detailed output is written in a binary unformatted form when requested on tape, disk, or punched on cards.

d) Card(s) # 4: Integer Constants (Optional)

FORMAT (I10,2X,15A4)

If the number of integer constants entered on card # 2, columns 9-10, is zero, no cards are to be inserted here. Otherwise, a card is inserted for each integer constant (up to 50 maximum) that is used. The data on each of these cards are entered as follows:

<u>Column #</u>	<u>Description</u>
1-10	Value of the integer constant.
13-72	Constant identification title.

e) Card(s) # 5: Deterministic Variables (Optional)

FORMAT(E10.0,2X,15A4)

If the number of deterministic variables entered on card # 2, columns 14-15, is zero, no cards are to be inserted here.

Otherwise a card is inserted for each deterministic variable (up to 50 maximum) that is used. The data on each of this set of cards are entered as follows:

<u>Column #</u>	<u>Description</u>
1-10	Value of the deterministic variable.
13-72	Variable identification title.

f) Card(s) # 6: Stochastic Variables:

FORMAT (F1.0,9X,15A4/3(7E10.0,10X))

Four cards are needed for each stochastic variable. The first of these four cards include the type of its distribution, whether it is discrete (code=0) or continuous (code=1), on the first column of the card. The identification title of the variable is written on columns 11 through 70 of this first card of the set. This title, when written on the output report, is broken into five fields; i.e., twelve characters on each line.

Accordingly, it is recommended that this title be broken into five logical parts. Each part should be centered in each of the five consecutive fields of twelve columns each.

The next three cards of this set include the values of the variable at the different 5% interval percentile points: 0%, 5%, ..., 100% of its cumulative distribution function, seven values per card. The data are entered on the three cards as follows:

1st Card:

<u>Column #</u>	<u>Description</u>
1	Distribution type: 0 = discrete, 1 = continuous.
10-70	Variable identification.

2nd Card:

<u>Column #</u>	<u>Description</u>
1-10	Value at 0% CDF.
11-20	Value at 5% CDF.
21-30	Value at 10% CDF.
31-40	Value at 15% CDF.
41-50	Value at 20% CDF.
51-60	Value at 25% CDF.
61-70	Value at 30% CDF.

3rd Card:

1-10	Value at 35% CDF.
11-20	Value at 40% CDF.
21-30	Value at 45% CDF.
31-40	Value at 50% CDF.
41-50	Value at 55% CDF.
51-60	Value at 60% CDF.
61-70	Value at 65% CDF.

4th Card:

1-10	Value at 70% CDF.
11-20	Value at 75% CDF.
21-30	Value at 80% CDF.
31-40	Value at 85% CDF.
41-50	Value at 90% CDF.
51-60	Value at 95% CDF.
61-70	Value at 100% CDF.

g) Card(s) # 7: Acceptability Criteria:

FORMAT (E10.0,2X,15A4)

Twenty-five acceptability criteria are allowed by GUSS for

discriminations that may be needed on the simulation data. If the number of acceptability criteria entered on card # 2, columns 24-25, is zero, no cards are to be inserted here. Otherwise, a card is inserted for each acceptability criterion (up to 25 maximum) that is needed. The data in each of these criteria are entered on a separate card as follows:

<u>Column #</u>	<u>Description</u>
1-10	Value of the criterion.
13-72	Criterion identification title.

h) Card(s) # 8: Column Titles:

FORMAT (15A4)

A card is needed for the title of each of the output variables (up to 7). The title of each column can be up to 60 characters long and starts at the first column of the card.

i) Card(s) # 9: Footnotes:

FORMAT (15A4)

These cards include all the footnotes to be printed after the output reports, one footnote per card starting at column # 1 and up to 60 characters long.

j) Cards # 10: Output Identification and Controls:

Up to 10 output reports can be generated by GUSS through different manipulation of the simulation results. Each of the requested reports will have a set of four kinds of control cards, followed by the set for the next report, etc.

First Card: FORMAT (2I5,2X,15A4)

This card includes the first line printed in the report, the last line printed, and the identification title of the report.

A block of output is specified for print in the report by specifying the outline of this block out of the whole results matrix which is printed with iterations as rows and output parameters as columns.

The data are entered on this card as follows:

<u>Column #</u>	<u>Description</u>
3- 5	1st row of printed block.
8-10	last row of printed block.
13-72	title of report, centered about column # 37 for better appearance on output report.

Second Card: FORMAT (2I5)

This card includes the number of different sorts required for this report and the number of footnotes to appear at its bottom. The data are entered as follows:

<u>Column #</u>	<u>Description</u>
4- 5	number of sorts (0-10).
9-10	number of footnotes (0-40).

Third Card Set: FORMAT (5I10)

These cards include the controls for performing the different sorts, one card for each sort. The different controls for each sort is entered on the respective card card as follows:

<u>Column #</u>	<u>Description</u>
7-10	The output variable used as the sort key.
17-20	The first output variable in the sorted block.
27-30	The last output variable in the sorted block.
37-40	The first iteration row where sorting starts.
47-50	The last iteration row where sorting ends.

GUSS allows up to 10 block sorts, descending order, for each output report. The results tableau can be sorted on any of the output parameters and for any requested length or number of iterations. If the block is to be printed as is without sorting, zero is entered on card # 2, columns 34-35, and no cards are to be inserted here.

Fourth Card Set: FORMAT (10I5)

These cards (0-4 cards) include the numbers of the footnotes to be printed at the bottom of the report. The numbers of the footnotes correspond to those desired from the list of footnotes as given in card(s) # 9. These numbers are entered 10 per card in columns 4-5, 9-10, etc., up to four cards. If no footnotes are desired (second card of cards # 10 has zero in second field), no cards are to be inserted here.

A.2.3 Linkage with "Model" Subroutine

Communication between GUSS and the problem model subroutine is made through the common storage area as follows:

- 1) Insert the following "COMMON" statement at the beginning of the subroutine:

```
COMMON/GUSS/KLAPS,LAPNUM,KONST(50),
      VECTOR(50),SAMPLE(50),ACCEPT(25),
      EXIT(7),RUN(10,20),KOUNT(10)
```

where:

KLAPS = total number of simulation iterations, read by GUSS.

LAPNUM = the iteration number at the time GUSS is passing the information to the subroutine.

KONST(I) = the vector of integer constants as read by GUSS (0 up to 50).

VECTOR(I) = the vector of deterministic variables and/or floating point constants as read by GUSS (0 up to 50).

SAMPLE(I) = the value of the ith random variable (in the order of their distributions as read by GUSS) generated by GUSS for that iteration and passed to the subroutine (1 to 50).

ACCEPT(I) = the ith acceptability criterion as read by GUSS
(0 to 25).

EXIT(I) = the value of the ith output parameter calculated
in the subroutine and passed to GUSS (1 to 7).

RUN(I,J) = the value calculated in the subroutine to be
printed next to the jth footnote of the ith report
(J=0 to 20 and I=1 to 10).

KOUNT(I) = the value of the ith counter as calculated in the
subroutine (0 to 10).

- 2) Each time GUSS passes information to the subroutine of the problem model, values will have been generated for each of the problem's random variables out of their distributions, and passed to the subroutine in the variables SAMPLE(I).
- 3) The subroutine will generate the output parameters, EXIT(I), which are the interaction, according to the problem's model, of the random values, deterministic variables and constants, and then passed to GUSS.
- 4) After the total number of iterations have been completed, GUSS will automatically tabulate the results, sorts, and produce them according to the output reports specifications as requested.

A.2.4 Sample Run

Figure 12 shows the subroutine for a simple investment model. This model allows for:

- One investment (stochastic) that is made at the beginning of the project period. This investment is depreciated according to a straight line depreciation method.
- One type of annual revenue (stochastic).
- One type of annual expenses (stochastic).
- A stochastic tax rate.

```

SUBROUTINE INMO
C
C   INMC1- SIMPLE INVESTMENT ANALYSIS MODEL
C
      COMMON/COSS/KLAPS,LAPNUM,KONST(50),VECTOR(50),SAMPLE(50),
1  ACCEPT(25),EXIT(7),FUN(10,20),KCOUNT(10)
C
C   STORE NUMBER OF INTEREST RATES IN   KONST(1)
C   IR= NUMBER OF INTEREST RATES
C   STORE INTEREST RATES IN VECTOR(1) ... VECTOR(IR)
C   STORE LIFE OF PROJECT IN VECTOR(IR+1)
C   STORE TAX LIFE OF EQUIPMENT IN VECTOR(IR+2)
C   STORE SALVAGE VALUE IN VECTOR(IR+3)
C   SAMPLE(1)=INVESTMENT
C   SAMPLE(2)=REVENUES
C   SAMPLE(3)=EXPENSES
C   SAMPLE(4)=TAX RATE
C   EXIT(1) ... EXIT(IR) = DISTRIBUTION FOR EACH GIVEN INTEREST RATE
C                           NET PRESENT VALUE
C
      IR=KONST(1)
      LIFE=VECTOR(IR+1)
      DEPN= (SAMPLE(1)-VECTOR(IR+3)) /VECTOR(IR+2)
      DO 100 I=1,IR
        EXIT(I)=0
        DO 200 NLIFE=1,LIFE
          200 EXIT(I)=EXIT(I)+((1.-SAMPLE(4))*(SAMPLE(2)-SAMPLE(3)-DEPN)+DEPN)
          1  *(1.+VECTOR(I  ))**(-NLIFE)
        100 EXIT(I)=EXIT(I)-SAMPLE(1)
C*****
C
      RETURN
      END

```

Figure 12. Simple Investment Model

Appendix B, pp. 155-163, shows the input data to the program. Appendix B, pp. 164-171, shows the output results. Four more dummy stochastic variables are put in the input data to demonstrate the output blocks produced by GUSS.

A.3 INMO User's Guide

INMO is a general investment analysis model that can be used, in conjunction with GUSS, to study the risk of an investment prospect. The INMO routine includes the mathematical relationships necessary to calculate the cash flows from the different components of an investment. It calculates the net present value of the investment for up to seven discounting rates of return per run.

The interaction between INMO and GUSS is made through the use of the variables: SAMPLE, KONST, etc., that are allowed by GUSS for input, and through the variable EXIT for the resultant net present values.

A.3.1 INMO's Capabilities

INMO can be applied to the study of a wide range of investment problems that may have any subset or all of the following variables:

- 1) Capital Investments:
 - a) Working Capital,
 - b) Land, and
 - c) Equipment and other depreciable items.

The following is allowed in the model:

- a) Investment items can be owned or leased.
- b) A time lag can be assumed before the investment item is installed--deterministic.
- c) A service life for owned items -- deterministic.

- d) A salvage value (as a percent of the original investment) at the end of its service life -- deterministic.
- e) INMO automatically replaces items that retire during the life of the project with the same kind items.
- f) INMO offers three depreciation methods. Straight line, Sum of the Year's Digits, and Double Rate Declining Balance.
- g) INMO accounts for tax credits resulting from depreciating the items.
- h) INMO assumes a recovery of a salvage value equal to the book value of the equipment at the end of its service life or the project's life, whichever occurs first.

2) Revenues from Sales:

INMO allows the following for each saleable product:

- a) A time lag until the product is introduced to the market -- stochastic.
- b) A growth period to maturity volumes (linear growth rates) -- stochastic.
- c) Maturity volume -- stochastic.
- d) A price per unit for each product -- stochastic.
- e) A direct cost per unit for each product -- stochastic.

3) Overhead Expenses:

The following is allowed for each expense item:

- a) A time lag until the expense is to be incurred -- stochastic.
- b) A growth period to the maturity value of the expense -- stochastic.
- c) Maturity value of the overhead expense -- stochastic.

4) Tax Rate:

A stochastic tax rate is allowed by INMO. All calculations for cash flows, and consequently the net present values, are made on an after tax basis.

5) Net Present Value:

INMO calculates and presents the distributions of up to seven NPV's calculated for up to seven discounting rates of return.

The current capabilities of INMO is constrained to a limited number of variables that are reasonable for most practical applications. These restrictions are imposed by the number of variables allowed by GUSS for computer core considerations. The constraints on the number of investment items, the number of products, and the number of overhead expenses are given by:

$$1) \quad V \leq 23$$

$$2) \quad 3*V+I \leq 49$$

$$3) \quad V+5*P+3*E \leq 49$$

where:

V = number of investment items,

I = number of discounting rates of return,

P = number of saleable products, and

E = number of overhead expenses.

Modifications can be made to the program to enable it to handle larger size problems as well as variations to the different assumptions in the model. The degree of ease or difficulty of such changes are dependent on the variance between the new problem and what INMO presently offers. Figure 13 gives a flow diagram of INMO.

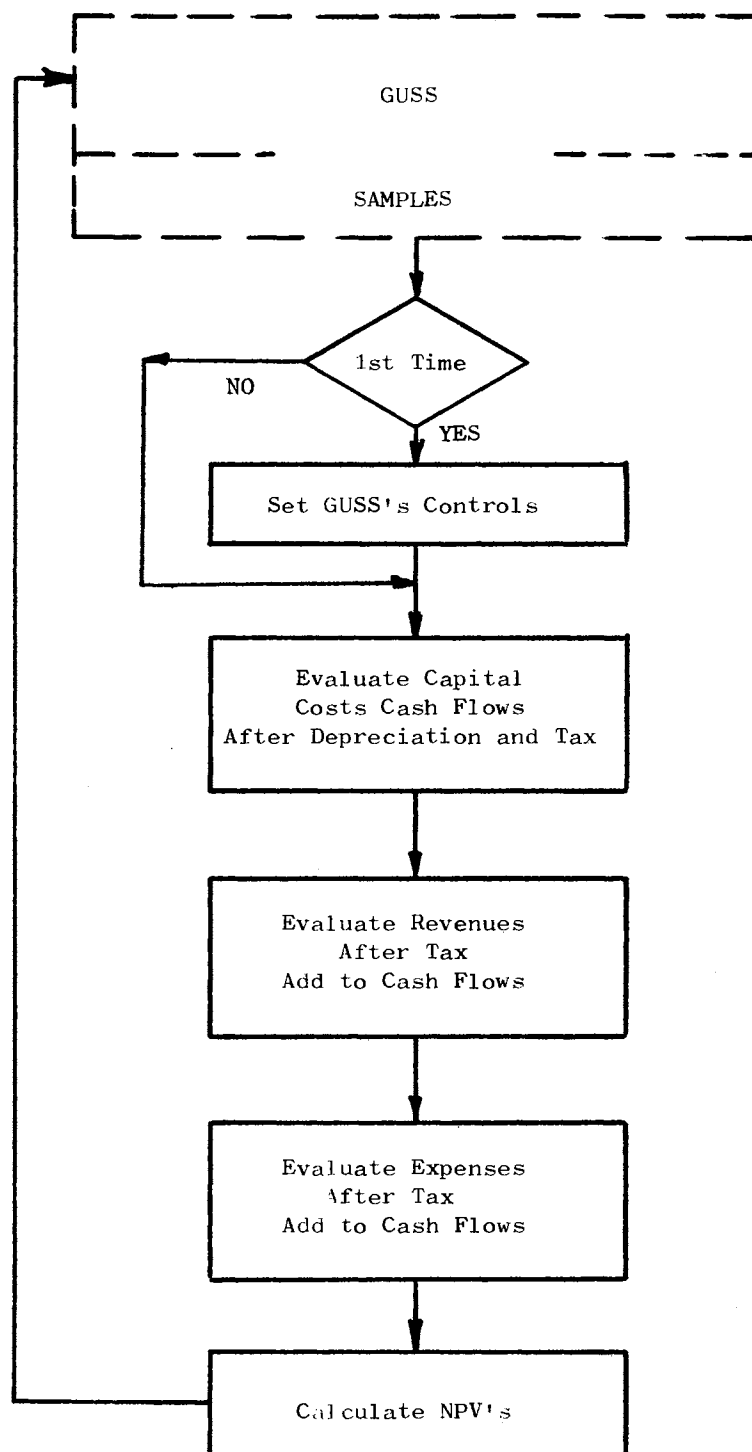


Figure 13. Investment Model (INMO) --
Flow Chart

A.3.2 Input Data Preparation

Appendix B, pp.172-179, gives the input sheets for INMO. The input is prepared as follows:

- a) Card # 1: Problem Identification Title

FORMAT (20A4)

This is the title of the investment problem under study. The title is written on the full 80 columns of the card and should be centered about column # 40 for better appearance on the program output.

- b) Card # 2: Problem Controls

FORMAT (10I5)

This card includes the count number of the problem controls as follows:

<u>Column #</u>	<u>Description</u>
2- 5	Number of simulation iterations -- maximum 1000.
9-10	Number of integer constants = 4 + 2 * number of investments.
14-15	Number of deterministic variables = 1 + number of discounting rates + 3 * number of investments.
19-20	Number of stochastic variables = number of investment items + 5 * number of products + 3 * number of overhead expenses.
29-30	Number of discounting rates of return.
34-35	Number of print out reports.
39-40	Total number of footnotes.

c) Card # 3: Output Devices

FORMAT (6I5)

These are the optional output devices for the detailed distributions of the resultant net present values. This card is filled as follows:

<u>Column #</u>	<u>Description</u>
5	Number of optional output devices to be used.
10	Code for 1st optional device used.
15	Code for 2nd optional output device used.
20	Code for 3rd optional output device used.
25	Code for 4th optional output device used.
30	Code for 5th optional output device used.

Available optional devices have the following codes:

<u>Code</u>	<u>Device</u>
1,2	disk files
3,4	tape files
6	printer
7	card punch

The detailed distribution output is written in a binary unformatted form when requested on tape, disk, or punched cards.

d) Cards 4-7: Investment Problem Parameters

FORMAT (I10,2X,15A4)

These cards are presented as follows:

<u>Card #</u>	<u>Column #</u>	<u>Description</u>
4	9-10	Number of investment items.
5	9-10	Number of products.
6	9-10	Number of overhead expenses.
7	9-10	Number of interest rates.

Columns 13-72 are prefilled on this sheet and are to be punched on the cards.

e) Cards Set # 8: Investment Parameters

FORMAT (I10,2X,15A4)

Two cards are needed for each investment item. The first card includes the code, in column 10, for whether the item is leased, code = 1, owned but not depreciable such as land and working capital, code = 2, or owned and depreciation allowed, code = 3. The second card indicates the code for the depreciation method to be used. Code "1" on this card indicates Straight Line Depreciation, code "2" indicates Double Rate Declining Balance, and code "3" indicates Sum of the Years Digits. If the first card has a code other than "3", the second card information about the depreciation method is redundant.

f) Card # 9: Life of Project

FORMAT (E10.0,2X,15A4)

This card includes the life of the project in columns 9 and 10. Columns 13-72 are prefilled and should be key-punched.

g) Cards Set # 10: Discounting Rates of Return

FORMAT (E10.0,2X,15A4)

These cards include the discounting rates of return, one per card. The value of the discounting rate of return, in

percentage, is filled in columns 1 through 10. Columns 13 through 72 are prefilled and are to be keypunched.

h) Card Set # 11: Investment Parameters

FORMAT (E10.0,2X,15A4)

This set of cards is composed of three card subsets, one subset for each investment. Each subset of three cards are filled as follows:

- 1) 1st card includes the time lag before investing, in columns 1 through 10.
- 2) 2nd card includes the service life of the investment item, in columns 1 through 10.
- 3) 3rd card includes the percentage salvage value at the end of the service life of the investment item, in columns 1 through 10.

Columns 13 through 72 of these three cards are prefilled and are to be keypunched.

i) Cards Set # 12: Tax

FORMAT (I1,9X,3(7E10.0,10X))

This set is composed of four cards about the distribution of the applicable tax rate. The first column of the first card of this set has the distribution code ("0" for discrete, "1" for continuous). Columns 11 through 70, divided into five fields of 12 columns each, include the tax title. This is prefilled and is to be keypunched. Each of the next three cards is divided into seven fields, ten columns each, to include the value of the tax at the different 5% intervals from 0 to 100%. The tax is entered in the field as a percentage value.

j) Cards Set # 13: Investments, Sales, and Overhead Expenses

FORMAT (I1,9X,15A4,3(7E10.0,10X))

This set of cards is divided into three major subsets.

1) The first one includes the distributions of the values of the different items.

2) The second subset has the distributions of the five parameters concerning the sales from each of the different products. These five parameters are:

- a) Selling price per unit of product.
- b) Direct cost per unit of product.
- c) Maturity volume of product sales.
- d) Time lag before selling starts.
- e) Sales growth period to maturity.

These groups are repeated for each of the different products.

3) The third subset has the distributions of the three parameters concerning overhead expenses as follows:

- a) Value of the expense at maturity.
- b) Time lag before incurring the expense.
- c) Growth period for the expense to mature.

The data for each of the above parameters are entered on four cards. The first card has information about the distribution code in column 1, ("0" for discrete and "1" for continuous), and the distribution title in columns 11 through 70, divided into five fields, 12 columns each, which is printed in five lines in the output report. The next three cards have the five percentile values of the distribution, seven per card, in the first seven fields of ten columns each.

k) Cards Set # 14: Output Column Titles

FORMAT (15A4)

This set of cards includes the titles of the different output columns, one per card, in the fields one through 60.

1) Cards Set # 15: Footnotes

FORMAT (15A4)

This set of cards includes the footnotes to be printed at the bottom of the different output reports, one per card, in the fields one through 60.

m) Cards Set # 16: Output Report Controls

Up to ten output reports are allowed by INMO. This set of cards include the different controls and information for each of these reports. Each set of cards for a report are ordered as follows:

1) 1st Card: Report Controls and Titles

FORMAT (2I5,2X,15A4)

<u>Column #</u>	<u>Description</u>
2- 5	1st line to be printed in report.
7-10	Last line to be printed in report.
13-72	Title of report, centered about column #41 for better appearance in output report.

2) 2nd Card: Number of Sorts and Footnotes

FORMAT (2I5)

<u>Column #</u>	<u>Description</u>
4- 5	Number of sorts required for report.
9-10	Number of footnotes to be printed at the end of the report.

3) 3rd Card Set: Sorts Controls

FORMAT (5I10)

<u>Column #</u>	<u>Description</u>
1-10	Output column used as sort key.
11-20	First output row for the start of sorting.
21-30	Last output row to end the sorting.
31-40	First output column in the sorted block.
41-50	Last output column in the sorted block.

A card, punched as above, is needed for each requested sort of those needed for a given report.

4) 4th Card Set: Report's Footnotes

The sequence numbers of the footnotes requested for the report are entered on these cards, ten per card, in the order they are to appear.

A.3.3 Sample Run

Refer to Chapter V, pp. 46-66, for a sample run.

A.4 RUM User's Guide

RUM is a program that calculates the utility of an investment according to the Farrer, Cramer and Smith, and/or Bussey's utility models, see section 3.3. The flow chart of RUM is given in Figure 14.

The coefficients of a requested utility model are entered to RUM together with other information necessary for the model, on the input form, Appendix B, p. 180. RUM gives the utility measure of the investment for the requested models.

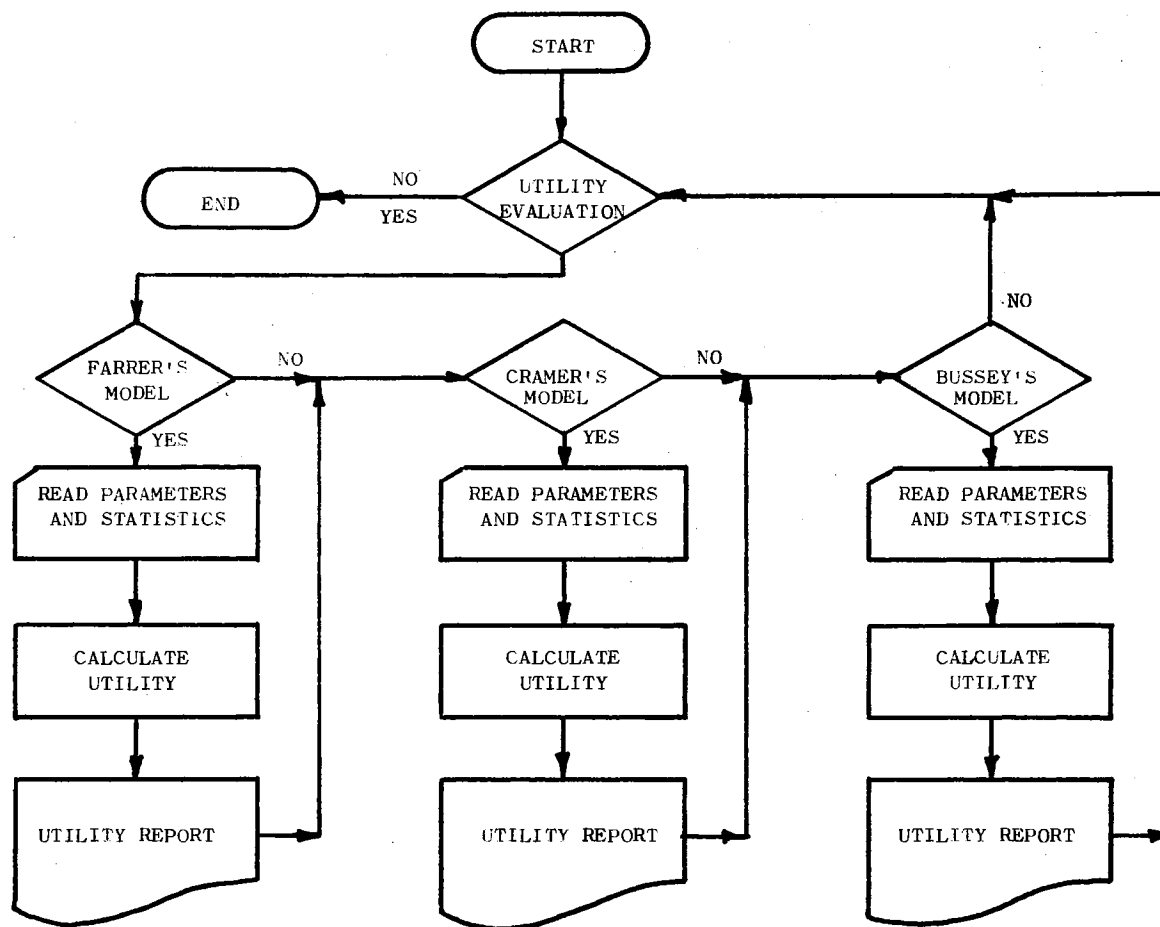


Figure 14. Routine for Utility Measurement (RUM)--Flow Chart

A.4.1 Input Data Preparation

A maximum of five cards are used to enter the data to RUM. The first card is the problem identification title. The second card is a control card indicating which of the three utility models is desired. The next three cards, one for each utility model, include the information needed for the model. If a model is not requested, the respective information card is not needed. RUM data cards are filled as follows:

- a) Card # 1: Problem Identification Title

FORMAT (20A4)

This is the title of the run. The title is written on the full 80 columns of the card. It should be centered about column # 40 for better appearance on the program output.

- b) Card # 2: Utility Model Control Card

FORMAT (3(I1,9X))

This card indicates whether or not a utility model is to run. This is indicated by entering "1" if the model is requested, and "0" otherwise. If the entered code is "1", a card with the utility model's data is to be inserted, but it should not be included otherwise. The fields on this card are divided as follows:

<u>Column #</u>	<u>Description</u>
1	Code for Farrer's model.
11	Code for Cramer and Smith's model.
12	Code for Bussey's model.

- c) Card # 3: Information for Farrer's Model

FORMAT (10E10.3)

Farrer's utility model is as follows:

$$u = 1 - e^{-aV}$$

where:

u = utility value,

a = constant, and

V = Net Present Value.

The information for a and V are entered on the card as follows:

<u>Column #</u>	<u>Description</u>
1-10	Value of "a".
11-20	Value of "V".

This card is inserted only if column # 1 on the control card has "1" in it.

d) Card # 4: Information for Cramer and Smith's Model

FORMAT (10E10.3)

The Cramer and Smith's model is given as follows:

$$u = \mu - a\sigma^b P^c$$

where:

u = utility value,

a = constant parameter,

b = constant parameter,

c = constant parameter,

μ = expected net present value,

σ = standard deviation of net present value, and

P = amount of investment.

The above data are entered on the card as follows:

<u>Column #</u>	<u>Description</u>
1-10	Value of "a".
11-20	Value of "b".

21-30	Value of "c".
31-40	Value of " μ ".
41-50	Value of " σ ".
51-60	Value of "P".

This card is inserted only if the Cramer and Smith's model is required, omit otherwise.

e) Card # 5: Information for Bussey's Model

The Bussey's model is given as follows:

$$u = a \cdot m_1 - b \cdot m_2 + c \cdot m_3 - d \cdot m_4$$

where

u = utility value,

a, b, c, d = constant parameters, and

m_i = the i th moment of the net present value distribution.

The above data are entered on the card as follows:

<u>Column #</u>	<u>Description</u>
1-10	Value of "a".
11-20	Value of "b".
21-30	Value of "c".
31-40	Value of "d".
41-50	Value of " m_1 ".
51-60	Value of " m_2 ".
61-70	Value of " m_3 ".
71-80	Value of " m_4 ".

This card is inserted only if the Bussey's model is requested, omit otherwise.

APPENDIX B

INPUT DATA SHEETS AND UPFAR

MODULES' INPUT/OUTPUT

B.1 PRODIP Input Data Sheets

01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
1ST CARD																																																																															
PROBLEM IDENTIFICATION TITLE																																																																															
2ND CARD																																																																															
CENTER TITLE AROUND CENTERLINE																																																																															
ENTER: 1 = Desired 0 = Skip																																																																															
PEARSON'S OPTION A =																																																																															
3RD CARD																																																																															
ENTER: 1 = Desired 0 = Skip																																																																															
CONTINUOUS OPTION B =																																																																															
EXPONENTIAL NORMAL LOGNORMAL FISHER-TIPPETT I FISHER-TIPPETT II WEIBULL BETA																																																																															
4TH CARD																																																																															
ENTER: 1 = Desired 0 = Skip																																																																															
DISCRETE OPTION C =																																																																															
POISSON TRUNCATED POISSON BINOMIAL NEGATIVE BINOMIAL TRUNCATED NEG. BINOMIAL																																																																															
5TH CARD																																																																															
ENTER: 1 = Desired 0 = Skip																																																																															
N NV H SHFE HY P LR																																																																															

PROBLEM

PAGE

— of

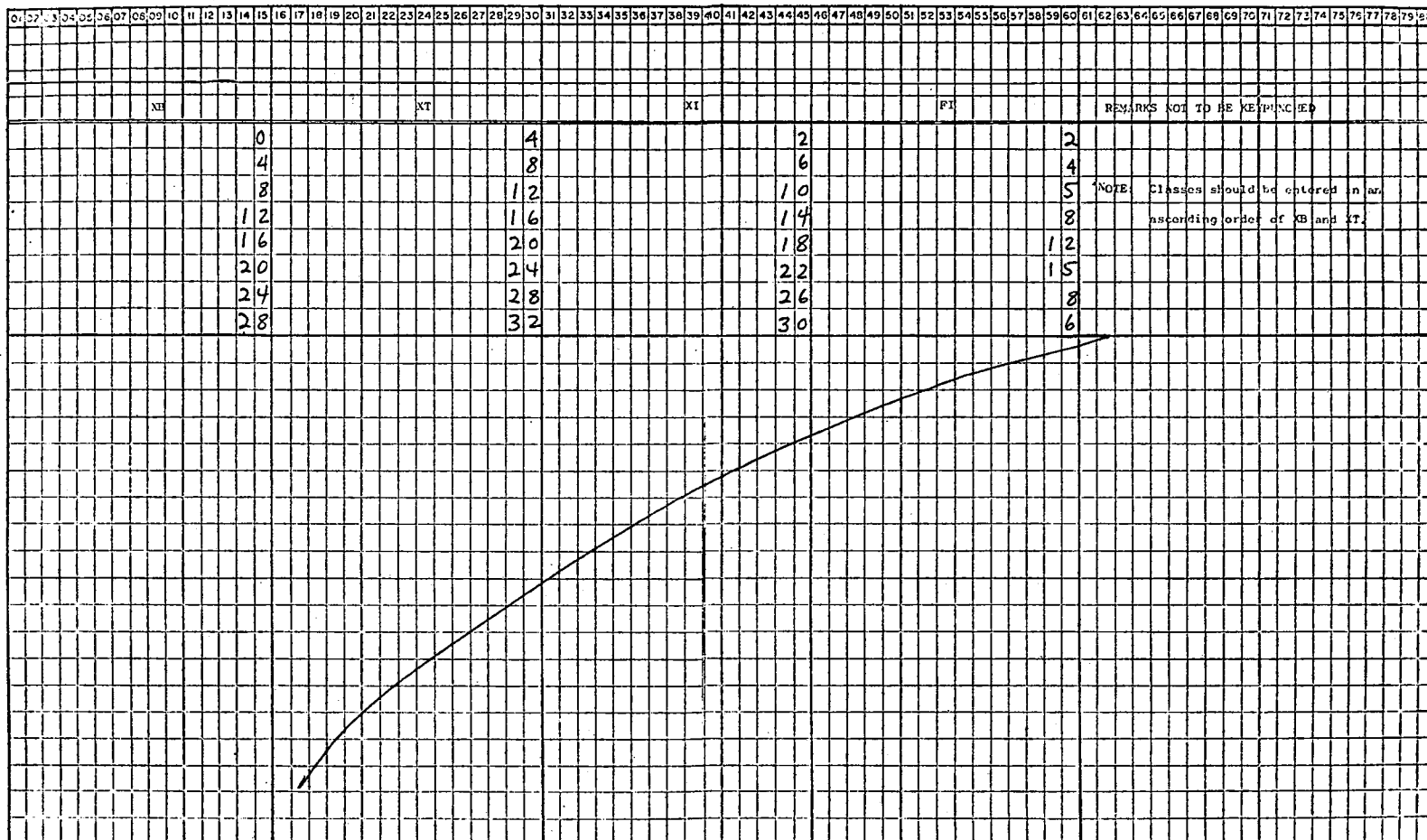
— DATE

— WRITTEN

01 02 03 04 05 06 07 08 09 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80																																																																															
XH										XT										XI										FI										REMARKS NOT TO BE KEYPUNCHED																																							
																																																												NOTE: Classes should be entered in an ascending order of XH and XT.																			

B.2 PRODIP Sample Run Input

126



PRODIP -- SAMPLE RUN

PRODIP -- SAMPLE RUN

PEARSCN SYSTEM

DISTRIBUTION TYPE 1

K = -0.10277359D 00
B1 = C.18359870D 00
B2 = 0.25630548D 01
LAMBDA= 0.28417515D 01
H = 0.0
N = 60

PM1 = 0.18733333D 02	PM2 = 0.40293333D 03	PM3 = 0.93357333D 04	PM4 = 0.22753173D 06	PM5 = 0.57610677D 07
PM6 = 0.15032657D 09	PM7 = 0.40188486D 10	PM8 = C.10958179D 12	M1 = 0.18733333D 02	M2 = 0.51995556D 02
S = C.72107944D 01	M3 = -C.16065126D 03	M4 = 0.69293155D 04	M5 = -0.49649077D 05	M6 = 0.12788734D 07
M8 = C.28082829D 09	BE1 = 0.18359870D 00	EE2 = 0.25630548D 01	BE3 = 0.10912655D 01	BE4 = 0.90976429D 01
BE6 = 0.38421646D 02	SIGB1= 0.16073571D 00	SIGB2= 0.35747700D 00	SIGL= 0.83530255D 01	SIGK= 0.78099880D 01
A = C.97014231D 01	B = 0.54273635D 01	C = 0.55630548D 01	D = 0.68520500D 01	E = 0.14246865D 01
F = -0.14246865D 01	C1 = 0.27136817D 01	C2 = -0.14246865D 01	SGC1= 0.20309702D 01	SGC2= 0.86237053D 00

XI

0.20000000D 01
C.60000000D 01
C.10000000D 02
0.14000000D 02
C.18000000D 02
C.22000000D 02
0.26000000D 02
C.30000000D 02

FI

0.20000000D 01
0.40000000D 01
0.50000000D 01
0.80000000D 01
0.12000000D 02
C.15000000D 02
0.80000000D 01
0.60000000D 01

SAMPLE STATISTICS

XE	XT	XI	FI	CFI	DFO
C.C	C.400000D 01	C.200000D 01	0.200000D 01	0.200000D 01	0.333333D-01
C.400000D 01	C.800000D 01	0.600000D 01	0.400000D 01	C.600000D 01	0.100000D 00
C.800000D 01	C.120000D 02	0.100000D 02	0.500000D 01	0.110000D 02	0.183333D 00
0.120000D 02	C.160000D 02	0.140000D 02	0.800000D 01	0.190000D 02	0.316667D 00
C.160000D 02	0.200000D 02	0.180000D 02	0.120000D 02	0.310000D 02	0.516667D 00
C.200000D 02	C.240000D 02	0.220000D 02	0.150000D 02	0.460000D 02	0.766667D 00
0.240000D 02	C.280000D 02	0.260000D 02	0.800000D 01	0.540000D 02	0.900000D 00
C.280000D 02	C.320000D 02	0.300000D 02	0.600000D 01	0.600000D 02	0.100000D 01

PRODIP -- SAMPLE RUN

EXPONENTIAL DISTRIBUTION

DENSITY FUNCTION $G(X)=\text{LAMBDA1} \exp(-\text{LAMBDA1} X)$, $X=C$, $\text{LAMBDA1}=0$

DISTRIBUTION FUNCTION $H(X)=1-\exp(-\text{LAMBDA1} XT)$

$\text{LAMBDA1}=1/M1$

H(X)	FE
0.19226652D 00	0.11535991D 02
0.34756663D 00	0.93180065D 01
0.47300773D 00	0.75264658D 01
0.57433070D 00	0.60793783D 01
0.65617266D 00	0.49105174D 01
0.72227915D 00	0.39663853D 01
0.77567557D 00	0.32037854D 01
0.81880565D 00	0.25878047D 01

MAX ABS DIFF (DFO-H(X))= 0.28967439D 00
LAMBDA1= 0.53380783D-01
CHISQ = 0.61006565D 02
NG(CHISQ)= 6 SMFE= 0.50000000D 01
P(EXCEEDING CHISQ)= 0.17821716D-11 DF= 4

PRCDIP -- SAMPLE RUN

NORMAL DISTRIBUTION

DENSITY FUNCTION $G(X)=1/S*\text{SQRT}(2\text{PI}) \text{ EXP}(-.5(X-M/S)**2),$

$-INFINITY+X+INFINITY, -INFINITY+M+INFINITY, S=0$

DISTRIBUTION FUNCTION $H(X)=1/S*\text{SQRT}(2\text{PI})*\text{INTEGRAL}(\text{EXP}(-(T-M)**2/2*S**2)DT)$

M=MEAN, S=STANDARD DEVIATION

Z	H(X)	FE
-0.20432330D C1	0.20514815D-01	0.12308889D 01
-0.14885091D 01	0.68308231D-01	0.28676050D 01
-0.93378524D 00	0.17520745D 00	0.64139530D 01
-0.37906133D 00	0.35232103D 00	0.10626815D 02
0.17566257D 00	0.56972042D 00	0.13043964D 02
0.72038647D 00	0.76742290D 00	0.11862149D 02
0.12851104D 01	0.90062326D 00	0.79920212D 01
0.18398343D 01	0.96710362D 00	0.39888218D 01

MAX ABS DIFF (DFC-H(X))= 0.53053755D-01

M= 0.18733333D 02

S= 0.72107944D 01

CHISQ= 0.19258176D 01

NG(CHISQ)= 5

SMFE= 0.50000000D 01

P(EXCEEDING CHISQ)= 0.38178074D 00

CF= 2

PRODIP -- SAMPLE RUN

SAMPLE STATISTICS LCGNORMAL DISTRIBUTION

YT	YI	FI	CFI	DF0
0.138629D 01	0.692147D 00	0.200000D 01	0.200000D 01	0.333333D-01
0.207944D 01	0.179176D 01	0.400000D 01	0.600000D 01	0.100000D 00
0.248491D 01	0.230259D 01	0.500000D 01	0.110000D 02	0.183333D 00
0.277259D 01	0.263906D 01	0.800000D 01	0.190000D 02	0.316667D 00
0.299573D 01	0.289037D 01	0.120000D 02	0.310000D 02	0.516667D 00
0.317805D 01	0.309104D 01	0.150000D 02	0.460000D 02	0.766667D 00
0.333220D 01	0.325810D 01	0.800000D 01	0.540000D 02	0.900000D 00
0.346574D 01	0.340120D 01	0.600000D 01	0.600000D 02	0.100000D 01

PRODIP -- SAMPLE RUN

LOGNORMAL DISTRIBUTION

DENSITY FUNCTION $G(Y)=1/S*\text{SQRT}(2\text{PI}) \text{ EXP}(-.5(Y-M/S)**2)$,

$-INFINITY+Y+INFINITY, -INFINITY+M+INFINITY, S=0$

DISTRIBUTION FUNCTION $H(X)=1/S*\text{SQRT}(2\text{PI})*\text{INTEGRAL}(\text{EXP}(-(T-M)**2/2*S**2)DT)$

$YI=\text{LN}(XI), YB=\text{LN}(XB), YT=\text{LN}(XT)$

$M=\text{MEAN}(YI), S=\text{STANDARD DEVIATION}(YI)$

Z	H(X)	FE
-0.24941567D 01	0.63127732D-02	0.37876639D 00
-0.12812791D 01	0.10004771D 00	0.56240960D 01
-0.57179123D 00	0.28373169D 00	0.11021039D 02
-0.68401554D-01	0.47273312D 00	0.11340086D 02
0.32205781D 00	0.62629566D 00	0.92137524D 01
0.64108634D 00	0.73926667D 00	0.67782604D 01
0.91082112D 00	0.81880507D 00	0.47723042D 01
0.11444760D 01	0.87378690D 00	0.32989098D 01

MAX ABS DIFF (DFC-H(X))= 0.15606645D 00

M= 0.28116795D 01 S= 0.57148982D 00

CHISQ= 0.19443438D 02

NG(CHISQ)= 6 SMFE= 0.50000000D 01

P(EXCEEDING CHISQ)= 0.22146895D-03

M2= 0.32660061D 00

H(Y)= 0.0

DF= 3

PRODIP -- SAMPLE RLN

FISHER-TIPPETT TYPE 1 DISTRIBUTION

DENSITY FUNCTION $G(X) = \text{ALPHA} \exp(-Y - \exp(-Y))$ WHERE
 $Y = \text{ALPHA}(X - U)$, $-\infty < X < \infty$, $\text{ALPHA} > 0$, $-\infty < U < \infty$

DISTRIBUTION FUNCTION $H(X) = \exp(-\exp(-Y))$

$U = \text{MODE}$, $\text{ALPHA} = \text{SCALE PARAMETER}$

H(X)	FE
0.13199275D-01	0.79195647D 00
0.78926440D-01	0.39436299D 01
0.22535242D 00	0.87879587D 01
0.41718973D 00	0.11507835D 02
0.59872677D 00	0.10892222D 02
0.74009542D 00	0.84821185D 01
0.83811547D 00	0.58812025D 01
0.90156723D 00	0.38071060D 01

MAX ABS DIFF (DFC-H(X))= 0.10052307D 00
ALPHA= 0.13328679D 00 U= 0.14991424D 02
CHISQ= 0.85802215D 01
AG(CHISQ)= 5 SMFE= 0.50000000D 01
P(EXCEEDING CHISQ)= 0.13703408D-01 DF= 2

PRODIP -- SAMPLE RUN

FISHER-TIPPETT TYPE 2 DISTRIBUTION

DENSITY FUNCTION $G(X) = \text{GAMMA} / B(X/B)^{**}(-\text{GAMMA}-1) \text{ EXP}(-(X/B)^{**}(-\text{GAMMA}))$

$X=C, B=C, \text{GAMMA}=C$

DISTRIBUTION FUNCTION $H(X) = \text{EXP}(-(X/B)^{**}(-\text{GAMMA}))$

$\text{GAMMA} = \text{SLOPE}, B = \text{SCALE PARAMETER}$

H(X)	FE
0.20030020D-01	0.12018012D 01
C.19203471D 00	0.10320282D 02
0.36931801D 00	0.10636958D 02
0.49844343D 00	0.77475252D 01
C.59013847D 00	0.55017028D 01
0.65684196D 00	0.40022089D 01
0.70685896D 00	0.30010204D 01
C.74542919D 00	0.23142139D 01

MAX ABS DIFF (DF0-H(X)) = 0.25457081D 00

GAMMA = 0.12448264D 01

B = 0.11962363D 02

CHISQ = 0.54895704D 02

NG(CHISQ) = 5

SMFE = 0.5000000D 01

P(EXCEEDING CHISQ) = 0.12010115D-11

DF = 2

PROCIP -- SAMPLE RUN

WEIBULL DISTRIBUTION

DENSITY FUNCTION $G(X) = (GAMMA/THETA)X^{**}(GAMMA-1) \exp(-X^{**}GAMMA/THETA)$

X GREATER THAN OR=0, GAMMA GREATER THAN 0, THETA GREATER THAN 0

DISTRIBUTION FUNCTION $H(X) = 1 - \exp(-X^{**}GAMMA/THETA)$

GAMMA=SHAPE PARAMETER, THETA=SCALE PARAMETER

H(X)	FE
0.89660201D-02	0.53796121D 00
0.62735462D-01	0.32261665D 01
0.18575184D 00	0.73809825D 01
0.37254134D 00	0.11207370D 02
0.58509305D 00	0.12753102D 02
0.77195977D 00	0.11212004D 02
0.89899346D 00	0.76220213D 01
0.96501444D 00	0.39612585D 01

MAX ABS DIFF (DFD-H(X))= 0.68426379D-01

GAMMA= 0.28467332D 01

THETA= 0.57457959D 04

CHISQ= 0.27482630D 01

NG(CHISQ)= 5

SMFE= 0.50000000D 01

P(EXCEEDING CHISQ)= 0.25305929D 00

CF= 2

PRODIP -- SAMPLE RUN

SAMPLE STATISTICS BETA DISTRIBUTION

XPB	XPI	XPT	FI	CFI	DFD
0.0	0.625000D-01	0.125000D 00	0.200000D 01	0.200000D 01	0.333333D-01
0.125000D 00	0.187500D 00	0.250000D 00	0.400000D 01	0.600000D 01	0.100000D 00
0.250000D 00	0.312500D 00	0.375000D 00	0.500000D 01	0.110000D 02	0.183333D 00
0.375000D 00	0.437500D 00	0.500000D 00	0.800000D 01	0.190000D 02	0.316667D 00
0.500000D 00	0.562500D 00	0.625000D 00	0.120000D 02	0.310000D 02	0.516667D 00
0.625000D 00	0.687500D 00	0.750000D 00	0.150000D 02	0.460000D 02	0.766667D 00
0.750000D 00	0.812500D 00	0.875000D 00	0.800000D 01	0.540000D 02	0.900000D 00
0.875000D 00	0.937500D 00	0.100000D 01	0.600000D 01	0.600000D 02	0.100000D 01

PRODIP -- SAMPLE RUN

BETA DISTRIBUTION

DENSITY FUNCTION $G(X) = (\text{GAMMA}(\text{LAMBDA} + \text{THETA}) / (\text{GAMMA}(\text{LAMBDA}) * (\text{GAMMA}(\text{THETA})))) * X^{**}(\text{LAMBDA} - 1) * (1 - X)^{**}(\text{THETA} - 1)$

X GREATER OR=0 AND X LESS THAN OR =1, LAMBDA GREATER THAN 0, THETA GREATER THAN 0

DISTRIBUTION FUNCTION $H(X) = (\text{GAMMA}(\text{LAMBDA} + \text{THETA}) / (\text{GAMMA}(\text{LAMBDA}) * (\text{GAMMA}(\text{THETA})))) \text{ INTEGRAL } (X^{**}(\text{LAMBDA} - 1) * (1 - X)^{**}(\text{THETA} - 1) DX)$

LAMBDA, THETA=SHAPE PARAMETERS

F(X)	FE
0.19996749D-01	0.11998049D 01
0.87600573D-01	0.40562535D 01
0.20168752D 00	0.68451925D 01
0.35474338D 00	0.91833521D 01
0.53463139D 00	0.10793281D 02
0.72393955D 00	0.11358490D 02
0.89662018D 00	0.10360838D 02
0.10000000D 01	0.62027891D 01

THETA= 0.15670463D 01 LAMBDA= 0.22127639D 01 M1= 0.58541667D 00 M2= 0.50776910D-01
MAX ABS DIFF (CFD-F(X))= 0.42727112D-01
CHISQ= 0.26021231D 01
NG(CHISQ)= 7 SMFE= 0.50000000D 01
P(EXCEEDING CHISQ)= 0.62644706D 00 DF= 4

PRODIP -- SAMPLE RUN

POISSON DISTRIBUTION

DENSITY FUNCTION $P(X) = \exp(-M) (M^X / X \text{ FACTORIAL})$ $X=0,1,2,***$ $M=0$

DISTRIBUTION FUNCTION $H(X) = \exp(-M) * \text{SUMMATION}(M^X / X \text{ FACTORIAL})$

$M = \text{MEAN}$

$F(X)$

FE

PRODIP -- SAMPLE RUN

NEGATIVE BINOMIAL DISTRIBUTION

(ESTIMATION OF PARAMETERS BY FIRST TWO MOMENT METHOD)

DENSITY FUNCTION $P(X) = (\text{GAMMA}(X+K1)/X \text{ FACTORIAL GAMMA}(K1)) * P^{**K1} * (1-P)^{**X}$

X=0,1,2,****

K GREATER 0 P GREATER OR =0 AND P LESS 1 OR =1

DISTRIBUTION FUNCTION $H(X) = \text{SUMMATION}(\text{GAMMA}(X+K1)/X \text{ FACTORIAL} * \text{GAMMA}(K1)) * P^{**K1} * (1-P)^{**X}$

H(X)

FE

0.52382465D-03	0.31429479D-01
0.99011675D-02	0.56264057D 00
0.42727107D-01	0.19695564D 01
0.96549031D-01	0.32293154D 01
0.15286585D 00	0.33790089D 01
0.19655403D 00	0.26212909D 01
0.22390770D 00	0.16412202D 01
0.23848282D 00	0.87450735D 00

MAX ABS DIFF (DFO-H(X)) = 0.76151718D 00

K = 0.10550641D 02

P = 0.36028720D 00

CHISQ = 0.15401869D 03

NG(CHISQ) = 2

SMFE = 0.50000000D 01

P(EXCEEDING CHISQ) = NO TEST

DF = -1

PRODIP -- SAMPLE RUN

TRUNCATED NEGATIVE BINOMIAL DISTRIBUTION

(ESTIMATION OF PARAMETERS BY BRASS METHOD)

DENSITY FUNCTION $P(X) = (\text{GAMMA}(K+X)/X \text{ FACTORIAL GAMMA}(K)) * (P^K * (1-P)^X / (1-P^K))$

X=1,2,3,****

K GREATER 0 P GREATER OR =0 AND P LESS 1 OR =1

DISTRIBUTION FUNCTION $H(X) = \text{SUMMATION}(\text{GAMMA}(K+X)/X \text{ FACTORIAL GAMMA}(K)) * (P^K * (1-P)^X / (1-P^K))$

H(X)	FE
0.63504717D-03	0.38102830D-01
0.10936294D-01	0.61807482D 00
0.44922275D-01	0.20391589D 01
0.98755473D-01	0.32299919D 01
0.15410512D 00	0.33209788D 01
0.19682307D 00	0.25630768D 01
0.22368483D 00	0.16117060D 01
0.23816704D 00	0.86893217D 00

MAX ABS DIFF (DFC-H(X))= 0.76183296D 00

K = 0.59598662D 01 P= 0.34827763D 00

CHISQ= 0.15617864D 03

NG(CHISQ)= 2 SMFE= 0.50000000D 01

P(EXCEEDING CHISQ)=NO TEST DF= -1

PRODIP -- SAMPLE RUN
 TRUNCATED POISSON DISTRIBUTION
 (MAXIMUM LIKELIHOOD ESTIMATION)
 DENSITY FUNCTION $P(X) = \exp(-M) * M^X / X \text{ FACTORIAL} (1 - \exp(-M))$
 $X=1,2,****$ M GREATER THAN 0
 DISTRIBUTION FUNCTION $H(X) = \exp(-M) * \text{SUMMATION } M^X / X \text{ FACTORIAL} * (1 - \exp(-M))$
 H(X) FE

B.4 GUSS Input Data Sheets

PROBLEM

PAGE

OF

DATE

— WRITTEN

01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
PROBLEM IDENTIFICATION TITLE																																																																															
KEEP SYMMETRICAL ABOUT CENTERLINE																																																																															
VARIABLE CONTROLS																																																																															
KLAPS								KONST								KDET								KXAND								KXNEPT								KXKIT								KBLK								KNOTS																							
OPTIMAL OUTPUT DEVICES FOR DETAILED DISTRIBUTIONS																																																																															
# OF DEVICES								1st								2nd								3rd								4th								5th								REMARKS NOT KEYPUNCHED																															
																																								CODES								1, 2 = DISKS								3, 4 = TAPES								6 = PRINTER								7 = PUNCH							

PROBLEM

PAGE

OF

DATE _____

WRITTEN

01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	
											<input type="checkbox"/> INTEGER CONSTANTS										<input type="checkbox"/> FLOATING DETERMINISTIC VARIABLES										<input type="checkbox"/> ACCEPTABILITY CRITERIA																																																	
VALUE											DESCRIPTION																																																																					

DATA SHEET

PROBLEM _____

PAGE _____

OF _____

DATE _____

WRITTEN _____

01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
TYPE										VARIABLE TITLE (Spread in 5 fields)																																																																					
0%										5%										10%										15%										20%										25%										30%																			
35%										40%										45%										50%										55%										60%										65%																			
70%										75%										80%										85%										90%										95%										100%																			
TYPE										VARIABLE TITLE (Spread in 5 fields)																																																																					
0%										5%										10%										15%										20%										25%										30%																			
35%										40%										45%										50%										55%										60%										65%																			
70%										75%										80%										85%										90%										95%										100%																			

PROBLEM

PAGE

OF

DATE _____

WRITTEN

COLUMN TITLES	FOOTNOTES
DESCRIPTION	

152

B.5 GUSS -- Sample Model Subroutine

```

SUBROUTINE INMO
C
C   INMC1- SIMPLE INVESTMENT ANALYSIS MODEL
C
      COMMON/COSS/KLAPS,LAPNUM,KONST(50),VECTOR(50),SAMPLE(50),
1  ACCEPT(25),EXIT(7),RUN(10,20),KCUNT(10)
C
C   STORE NUMBER OF INTEREST RATES IN   KONST(1)
C   IR= NUMBER OF INTEREST RATES
C   STORE INTEREST RATES IN VECTOR(1) ... VECTOR(IR)
C   STORE LIFE OF PROJECT IN VECTOR(IR+1)
C   STORE TAX LIFE OF EQUIPMENT IN VECTOR(IR+2)
C   STORE SALVAGE VALUE IN VECTOR(IR+3)
C   SAMPLE(1)=INVESTMENT
C   SAMPLE(2)=REVENUES
C   SAMPLE(3)=EXPENSES
C   SAMPLE(4)=TAX RATE
C   EXIT(1) ... EXIT(IR) = DISTRIBUTION FOR EACH GIVEN INTEREST RATE
C                           NET PRESENT VALUE
C
      IR=KONST(1)
      LIFE=VECTOR(IR+1)
      DEPN= (SAMPLE(1)-VECTOR(IR+3)) /VECTOR(IR+2)
      DO 100 I=1,IR
        EXIT(I)=0
        DO 200 NLIFE=1,LIFE
          200 EXIT(I)=EXIT(I)+((1.-SAMPLE(4))*(SAMPLE(2)-SAMPLE(3)-DEPN)+DEPN)
          1      *(1.+VECTOR(I))**(-NLIFE)
        .100 EXIT(I)=EXIT(I)-SAMPLE(1)
C*****
C
      RETURN
      END

```

B.6 GUSS -- Sample Model Subroutine Input

PROBLEM GUS - Sample Run

PAGE 1 OF 8 DATE 10/20/73 WRITTEN SS

156

DATA SHEET

PROBLEM

GUSS - Sample Run

PAGE

2

OF

8

DATE

10/20/73

WRITTEN

SE

01 02 03 04 05 06 07 08 09 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80																																
<input checked="" type="checkbox"/> INTEGER CONSTANTS											<input checked="" type="checkbox"/> FLOATING DETERMINISTIC VARIABLES											<input checked="" type="checkbox"/> ACCEPTABILITY CRITERIA										
VALUE	DESCRIPTION																															
6	NUMBER OF INTEREST RATES	Integer																														
0.05	INTEREST RATES:	Floating																														
0.09																																
0.10																																
0.15																																
0.20																																
0.25																																
20.0	LIFE OF PROJECT																															
20.0	TAX LIFE																															
0.0	SALVAGE VALUE																															
		no accept. criteria																														

158

PROBLEM

PAGE 4

OF 8

DATE _____

10/20/73

WRITTEN

8

159

160

01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80										
TYPE										VARIABLE TITLE (Spread in 5 fields)																																																																															
01										DUMMY3																																																																															
0%										5%										10%										15%										20%										25%										30%																													
3000.										3000.										3000.										3000.										3000.										3000.										3000.																													
35%										40%										45%										50%										55%										60%										65%																													
3600.										3000.										3000.										3000.										3000.										3000.										3000.																													
70%										75%										80%										85%										90%										95%										100%																													
3000.										3000.										3000.										3000.										3000.										3000.										3000.																													
X										X										X										X										X										X										X										X																			
TYPE										VARIABLE TITLE (Spread in 5 fields)																																																																															
01										DUMMY4																																																																															
0%										5%										10%										15%										20%										25%										30%																													
0.5										0.5										0.5										0.5										0.5										0.5										0.5																													
35%										40%										45%										50%										55%										60%										65%																													
0.5										0.5										0.5										0.5										0.5										0.5										0.5																													
70%										75%										80%										85%										90%										95%										100%																													
0.5										0.5										0.5										0.5										0.5										0.5										0.5																													

162

DATA SHEET

PROBLEM

GUSS Sample Run

PAGE 8

OF

8

DATE

10/20/73

WRITTEN

SL

REPORT CONTROLS AND TITLE		MINBLK		MAXBLK		TITLE OF REPORT	
1	50	NET PRESENT VALUE AT 5, 10, 15, 20, AND 25 PERCENT					
CENTER TITLE AROUND CENTERLINE							
REPORT SORTING AND FOOTNOTES CONTROLS		ASORT		AFOOT			
6	2						
REPORTING SORTING CONTROLS							
ASX	MINROW	MAXROW	MINCOL	MAXCOL	REMARKS NOT KEYPUNCHED		
1	1	1	1	50	MINBLK 1 MAXBLK 50		
2	2	2	1	50	MINROW 1		
3	3	3	1	50	KEYROW 1		
4	4	4	1	50	MAXROW 1		
5	5	5	1	50	LAP 1 MINCOL 1 MAXCOL 50 LAP 1000		
6	6	6	1	50			
REMARKS NOT KEYPUNCHED							
1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8
FOOTNOTES 1 through 10							
FOOTNOTES 11 through 20							
FOOTNOTES 21 through 30							
FOOTNOTES 31 through 40							

B.7 GUSS Sample Run Output

```
*****
*
*
*
*   G U S S   -   -   S A M P L E   R U N
*
*
*
*****
```

INTEGER CONSTANTS & INDEXES

NUMBER OF INTEREST RATES

= 6

DETERMINISTIC VARIABLES

INTEREST RATES

0.50000E-01

0.90000E-01

0.10000E 00

0.15000E 00

0.20000E 00

0.25000E 00

LIFE OF PROJECT

0.20000E 02

TAX LIFE

0.20000E 02

SALVAGE VALUE

0.0

INPUT RANDOM VARIABLES

CUM.PCT	INVESTMENT	REVENUES	EXPENSES	TAX RATE	DUMMY1	DUMMY2
0.	0.70000E 05	0.15000E 05	0.30000E 04	0.50000E 00	0.70000E 05	0.15000E 05
5.	0.70000E 05	0.15000E 05	0.30000E 04	0.50000E 00	0.70000E 05	0.15000E 05
10.	0.80000E 05	0.15200E 05	0.30000E 04	0.50000E 00	0.80000E 05	0.15200E 05
15.	0.80000E 05	0.15300E 05	0.30000E 04	0.50000E 00	0.80000E 05	0.15300E 05
20.	0.80000E 05	0.15400E 05	0.30000E 04	0.50000E 00	0.80000E 05	0.15400E 05
25.	0.80000E 05	0.15500E 05	0.30000E 04	0.50000E 00	0.80000E 05	0.15500E 05
30.	0.80000E 05	0.16000E 05	0.30000E 04	0.50000E 00	0.80000E 05	0.16000E 05
35.	0.10000E 06	0.16500E 05	0.30000E 04	0.50000E 00	0.10000E 06	0.16500E 05
40.	0.10000E 06	0.17000E 05	0.30000E 04	0.50000E 00	0.10000E 06	0.17000E 05
45.	0.10000E 06	0.17800E 05	0.30000E 04	0.50000E 00	0.10000E 06	0.17800E 10
50.	0.10000E 06	0.18500E 05	0.30000E 04	0.50000E 00	0.10000E 06	0.18500E 05
55.	0.11000E 06	0.19500E 05	0.30000E 04	0.50000E 00	0.11000E 06	0.19500E 05
60.	0.11000E 06	0.20000E 05	0.30000E 04	0.50000E 00	0.11000E 06	0.20000E 05
65.	0.11000E 06	0.22500E 05	0.30000E 04	0.50000E 00	0.11000E 06	0.22500E 05
70.	0.12000E 06	0.23000E 05	0.30000E 04	0.50000E 00	0.12000E 06	0.23000E 05
75.	0.12000E 06	0.23500E 05	0.30000E 04	0.50000E 00	0.12000E 06	0.23500E 05
80.	0.12000E 06	0.24000E 05	0.30000E 04	0.50000E 00	0.12000E 06	0.24000E 05
85.	0.12500E 06	0.24000E 05	0.30000E 04	0.50000E 00	0.12500E 06	0.24000E 05
90.	0.12500E 06	0.24300E 05	0.30000E 04	0.50000E 00	0.14000E 06	0.24300E 05
95.	0.14000E 06	0.24700E 05	0.30000E 04	0.50000E 00	0.12500E 06	0.24700E 05
100.	0.14000E 06	0.25000E 05	0.30000E 04	0.50000E 00	0.14000E 06	0.25000E 05

DIST. TYPE 1. 1. 0. 0. 1. 1.

0=DISCRETE, 1=CONTINUOUS

INPLT RANDCM VARIABLES

CON.PCT	DUMMY3	DUMMY4
C.	C.30000E C4	G.50000E C0
5.	0.30000E 04	0.50000E 00
10.	C.30000E 04	0.50000E 00
15.	C.30000E C4	C.50000E 00
20.	C.30000E 04	0.50000E C0
25.	C.30000E 04	0.50000E 00
30.	C.30000E C4	0.50000E C0
35.	0.30000E 04	0.50000E 00
40.	C.30000E C4	0.50000E 00
45.	0.30000E 04	0.50000E C0
50.	0.30000E 04	0.50000E 00
55.	C.30000E C4	C.50000E 00
60.	0.30000E 04	0.50000E C0
65.	0.30000E 04	0.50000E 00
70.	C.30000E C4	C.50000E 00
75.	0.30000E 04	0.50000E C0
80.	C.30000E 04	0.50000E 00
85.	C.30000E C4	0.50000E C0
90.	0.30000E 04	0.50000E 00
95.	C.30000E 04	0.50000E 00
100.	C.30000E C4	0.50000E C0

DIST. TYPE	C.	C.
------------	----	----

0=DISCRETE, 1=CONTINUOUS

COLUMN TITLES :

COLUMN 1 SHOWS THE NPV AT 5 PERCENT INTEREST RATE
COLUMN 2 SHOWS THE NPV AT 9 PERCENT INTEREST RATE
COLUMN 3 SHOWS THE NPV AT 10 PERCENT INTEREST RATE
COLUMN 4 SHOWS THE NPV AT 15 PERCENT INTEREST RATE
COLUMN 5 SHOWS THE NPV AT 20 PERCENT INTEREST RATE
COLUMN 6 SHOWS THE NPV AT 25 PERCENT INTEREST RATE

PCT.	1	2	3	4	5	6
100.0000	0.81713E 05	0.38454E 05	0.30474E 05	0.21048E 04	-0.13904E 05	-0.24453E 05
98.0000	0.75014E 05	0.36477E 05	0.28630E 05	0.12224E 04	-0.16811E 05	-0.28694E 05
96.0000	0.75778E 05	0.35157E 05	0.28073E 05	-0.13331E 03	-0.17866E 05	-0.29550E 05
94.0000	0.73560E 05	0.34107E 05	0.26420E 05	-0.17583E 04	-0.19130E 05	-0.30577E 05
92.0000	0.73449E 05	0.31413E 05	0.23659E 05	-0.47658E 04	-0.21684E 05	-0.32650E 05
90.0000	0.69244E 05	0.29320E 05	0.21956E 05	-0.50406E 04	-0.22290E 05	-0.33416E 05
88.0000	0.66829E 05	0.27552E 05	0.20306E 05	-0.62534E 04	-0.22627E 05	-0.33836E 05
86.0000	0.63667E 05	0.19885E 05	0.11809E 05	-0.17797E 05	-0.32043E 05	-0.39362E 05
84.0000	0.59561E 05	0.14153E 05	0.57760E 04	-0.19622E 05	-0.32131E 05	-0.39475E 05
82.0000	0.52449E 05	0.80548E 04	0.21226E 04	-0.20935E 05	-0.33028E 05	-0.41861E 05
80.0000	0.51792E 05	0.60935E 04	-0.44738E 03	-0.20986E 05	-0.34498E 05	-0.43054E 05
78.0000	0.51385E 05	0.58373E 04	-0.24580E 04	-0.21511E 05	-0.36048E 05	-0.45755E 05
76.0000	0.45741E 05	0.55389E 04	-0.26405E 04	-0.24929E 05	-0.37824E 05	-0.48074E 05
74.0000	0.48378E 05	0.52992E 04	-0.29075E 04	-0.25787E 05	-0.41043E 05	-0.48369E 05
72.0000	0.46146E 05	0.49176E 04	-0.29171E 04	-0.29925E 05	-0.43859E 05	-0.56331E 05
70.0000	0.42745E 05	0.43763E 04	-0.29187E 04	-0.31574E 05	-0.48247E 05	-0.57979E 05
68.0000	0.40212E 05	0.33361E 04	-0.33292E 04	-0.33476E 05	-0.48987E 05	-0.60460E 05
66.0000	0.36450E 05	0.20239E 04	-0.33514E 04	-0.33661E 05	-0.51811E 05	-0.60873E 05
64.0000	0.36345E 05	0.19583E 04	-0.49732E 04	-0.33715E 05	-0.52347E 05	-0.62780E 05
62.0000	0.34265E 05	-0.93644E 03	-0.62630E 04	-0.33804E 05	-0.52873E 05	-0.64659E 05
60.0000	0.32448E 05	-0.21273E 04	-0.95185E 04	-0.33920E 05	-0.53032E 05	-0.64714E 05
58.0000	0.30268E 05	-0.29824E 04	-0.10405E 05	-0.35430E 05	-0.53129E 05	-0.65413E 05
56.0000	0.28758E 05	-0.69713E 04	-0.11891E 05	-0.38058E 05	-0.54160E 05	-0.65481E 05
54.0000	0.28644E 05	-0.86419E 04	-0.15569E 05	-0.40573E 05	-0.54207E 05	-0.65496E 05
52.0000	0.28229E 05	-0.87479E 04	-0.15750E 05	-0.40748E 05	-0.55988E 05	-0.65625E 05
50.0000	0.27937E 05	-0.96639E 04	-0.16941E 05	-0.41077E 05	-0.56542E 05	-0.65809E 05
48.0000	0.27181E 05	-0.59292E 04	-0.17345E 05	-0.44139E 05	-0.57403E 05	-0.65862E 05
46.0000	0.23326E 05	-0.12352E 05	-0.19657E 05	-0.44526E 05	-0.57487E 05	-0.66125E 05
44.0000	0.19698E 05	-0.14068E 05	-0.20551E 05	-0.45084E 05	-0.57891E 05	-0.66144E 05
42.0000	0.19247E 05	-0.15327E 05	-0.21705E 05	-0.45246E 05	-0.58280E 05	-0.66392E 05
40.0000	0.17314E 05	-0.17541E 05	-0.24021E 05	-0.45353E 05	-0.58609E 05	-0.66579E 05
38.0000	0.14854E 05	-0.18533E 05	-0.25527E 05	-0.45873E 05	-0.59454E 05	-0.68994E 05
36.0000	0.11219E 05	-0.20148E 05	-0.25673E 05	-0.46374E 05	-0.59497E 05	-0.71402E 05
34.0000	0.90140E 04	-0.20304E 05	-0.25791E 05	-0.46795E 05	-0.61283E 05	-0.71780E 05
32.0000	0.88006E 04	-0.21062E 05	-0.26380E 05	-0.47362E 05	-0.63491E 05	-0.72325E 05
30.0000	0.77663E 04	-0.21220E 05	-0.27060E 05	-0.51362E 05	-0.64381E 05	-0.72960E 05
28.0000	0.67696E 04	-0.21792E 05	-0.27634E 05	-0.51485E 05	-0.64895E 05	-0.73526E 05
26.0000	0.67481E 04	-0.22407E 05	-0.27874E 05	-0.51796E 05	-0.66116E 05	-0.74718E 05
24.0000	0.66703E 04	-0.24482E 05	-0.30244E 05	-0.52268E 05	-0.66546E 05	-0.77215E 05
22.0000	0.59293E 04	-0.24752E 05	-0.30548E 05	-0.52474E 05	-0.67306E 05	-0.78473E 05
20.0000	0.21021E 04	-0.26698E 05	-0.32108E 05	-0.54145E 05	-0.71907E 05	-0.81526E 05
18.0000	0.12671E 04	-0.28541E 05	-0.34029E 05	-0.56032E 05	-0.72615E 05	-0.81815E 05
16.0000	0.12069E 04	-0.31172E 05	-0.37157E 05	-0.59092E 05	-0.72971E 05	-0.82789E 05
14.0000	0.35713E 03	-0.31835E 05	-0.37788E 05	-0.59549E 05	-0.74397E 05	-0.83602E 05
12.0000	-0.26898E 04	-0.34392E 05	-0.40240E 05	-0.61661E 05	-0.74675E 05	-0.83879E 05
10.0000	-0.48176E 04	-0.35368E 05	-0.41003E 05	-0.61677E 05	-0.74893E 05	-0.84438E 05
8.0000	-0.61501E 04	-0.36605E 05	-0.42223E 05	-0.62817E 05	-0.75514E 05	-0.84711E 05
6.0000	-0.64229E 04	-0.36990E 05	-0.42629E 05	-0.63299E 05	-0.76042E 05	-0.86959E 05
4.0000	-0.19349E 05	-0.51624E 05	-0.57578E 05	-0.79402E 05	-0.92856E 05	-0.10172E 06
2.0000	-0.21608E 05	-0.53279E 05	-0.59121E 05	-0.80536E 05	-0.93739E 05	-0.10244E 06

SUMMARY STATISTICS

DISTRIBUTION SUMMARY

PCT.	1	2	3	4	5	6
100.0000	0.81713E 05	0.38454E 05	0.30474E 05	0.21048E 04	-0.13904E 05	-0.24453E 05
90.0000	0.69244E 05	0.29320E 05	0.21956E 05	-0.50406E 04	-0.22290E 05	-0.33416E 05
80.0000	0.51792E 05	0.60935E 04	-0.44738E 03	-0.20986E 05	-0.34498E 05	-0.43054E 05
70.0000	0.42745E 05	0.43763E 04	-0.29187E 04	-0.31574E 05	-0.48247E 05	-0.57979E 05
60.0000	0.32448E 05	-0.21273E 04	-0.95185E 04	-0.33920E 05	-0.53032E 05	-0.64714E 05
50.0000	0.27937E 05	-0.56639E 04	-0.16541E 05	-0.41077E 05	-0.56542E 05	-0.65809E 05
40.0000	0.17314E 05	-0.17541E 05	-0.24021E 05	-0.45353E 05	-0.58609E 05	-0.66579E 05
30.0000	0.77663E 04	-0.21220E 05	-0.27060E 05	-0.51362E 05	-0.64381E 05	-0.72960E 05
20.0000	0.21021E 04	-0.26698E 05	-0.32010E 05	-0.54145E 05	-0.71907E 05	-0.81526E 05
10.0000	-0.48176E 04	-0.35368E 05	-0.41003E 05	-0.61677E 05	-0.74893E 05	-0.84438E 05
2.0000	-0.21608E 05	-0.53279E 05	-0.59121E 05	-0.80536E 05	-0.93739E 05	-0.10244E 06

DISTRIBUTION PARAMETERS

MEAN	0.28748E 05	-0.71299E 04	-0.13749E 05	-0.38010E 05	-0.52967E 05	-0.62822E 05
ST. DEV.	0.27026E 05	0.22728E 05	0.22061E 05	0.20066E 05	0.19257E 05	0.18926E 05

FOOTNOTES

N.B.

N.B. EACH COLUMN IS SORTED TO GIVE CDF

B.8 INMO Input Data Sheets

DATA SHEET

PROBLEM

PAGE

OF

DATE

WRITTEN

01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80																												
PROBLEM IDENTIFICATION TITLE																																																																																																											
KEEP SYMMETRICAL ABOUT CENTERLINE																																																																																																											
VARIABLES CONTROL																																																																																																											
KLAPS																																									KONST				KDET				KXAND				KXKEPT				KEXIT				KBLK				KNOTES																																										
OPTIONAL OUTPUT DEVICES FOR DETAILED DISTRIBUTION																																																																																																											
# OF DEVICES		1st				2nd				3rd				4th				5th				REMARKS NOT KEYPUNCHED																																																																																					
																		CODES 1,4 = DISKS 3,4 = TAPES 6 = PRINTER 7 = PUNCH																																																																																									
NUMBER OF		DESCRIPTION																																																																																																									
		NUMBER OF INVESTMENT ITEMS																																																																																																									
		NUMBER OF PRODUCTS																																																																																																									
		NUMBER OF OVERHEAD EXPENSES																																																																																																									
		NUMBER OF INTEREST RATES																																																																																																									

PROBLEM _____ PAGE _____ OF _____ DATE _____ WRITTEN _____

174

DATA SHEET

PROBLEM _____

PAGE _____

OF _____

DATE _____

WRITTEN _____

01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
PROJECT LIFE AND DISCOUNT RATES																																																																															
FLOATING DETERMINISTIC VARIABLES																																																																															
VALUE																				DESCRIPTION																																																											
																				LIFE OF PROJECT																																																											
																				1st DISCOUNT RATE																																																											
																				2nd DISCOUNT RATE																																																											
																				3rd DISCOUNT RATE																																																											
																				4th DISCOUNT RATE																																																											
																				5th DISCOUNT RATE																																																											
																				6th DISCOUNT RATE																																																											
																				7th DISCOUNT RATE																																																											

PROBLEM

PAGE

OF

DATE _____

WRITTEN

176

DATA SHEET

PROBLEM

PAGE

OF

DATE _____

WILFEN

01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
										<input type="checkbox"/> COLUMN TITLES																				<input type="checkbox"/> FOOTNOTES																																																	
																				DESCRIPTION																																																											

PROBLEM

PAGE

DE

DATE _____

... WRITTEN

179

B.9 RUM Input Data Sheet

DATA SHEET

PROBLEM

PAGE

OF

DATE

WRITTEN

01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
																																								CASE IDENTIFICATION TITLE																																							
REQUESTED UTILITY MODEL																																								1 = REQUESTED 0 = NOT REQUESTED																																							
FARRER										CRAMER AND SMITH										BUSSEY																																																											
/ / / / / / / / / /										/ / / / / / / / / /										/ / / / / / / / / /																																																											
FARRER: $U = 1 - \exp(-A \cdot V)$																																																																															
A										V																																																																					
CRAMER AND SMITH: $U = V - A \cdot (S \cdot P) \cdot C$																																																																															
A										B										C										V										S										P																													
BUSSEY: $U = A \cdot V_1 - B \cdot V_2 + C \cdot V_3 - D \cdot V_4$																																																																															
A										B										C										D										V1										V2										V3										V4									

B.10 The INCO Company Investment Problem--

GUSS Input

PROBLEM

- PAGE

DATE _____

WRITTEN

184

PROBLEM

PAGE

08

17

- DATE _____

10/20/73

WRITTEN

85

Курбанов

VALUE	DESCRIPTION	
0	TIME LAG FOR INVESTMENT	INVESTMENT: WORKING CAPITAL
20	SERVICE LIFE	
100	SALVAGE AT END OF SERVICE (AS % INVESTMENT)	
0	TIME LAG FOR INVESTMENT	INVESTMENT: LAND
20	SERVICE LIFE	
100	SALVAGE AT END OF SERVICE (AS % INVESTMENT)	
1	TIME LAG FOR INVESTMENT	INVESTMENT: BUILDINGS
25	SERVICE LIFE	
30	SALVAGE AT END OF SERVICE (AS % INVESTMENT)	
2	TIME LAG FOR INVESTMENT	INVESTMENT: HEAVY EQUIPMENT
10	SERVICE LIFE	
20	SALVAGE AT END OF SERVICE (AS % INVESTMENT)	
3	TIME LAG FOR INVESTMENT	INVESTMENT: LIGHT EQUIPMENT
5	SERVICE LIFE	
10	SALVAGE AT END OF SERVICE (AS % INVESTMENT)	

← keep punch →

DATA SHEET

PROBLEM

INCO - GUS

PAGE

6

OF

17

DATE

10/20/73

WRITTEN

82

01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80																																																												
TYPE										VARIABLE TITLE (Spread in 5 fields)																																																																																																																																	
1										LAND INVESTMENT																																																																																																																																	
0%										5%										10%										15%										20%										25%										30%																																																																															
12000.										12750.										13500.										14250.										15000.										15300.										15600.																																																																															
35%										40%										45%										50%										55%										60%										65%																																																																															
15900.										16200.										16500.										16800.										17100.										17400.										17700.																																																																															
75%										75%										80%										85%										90%										95%										100%																																																																															
18000.										18333.										18667.										19000.										19333.										19667.										20000.																																																																															
TYPE										VARIABLE TITLE (Spread in 5 fields)																																																																																																																																	
1										BUILDINGS INVESTMENT																																																																																																																																	
0%										5%										10%										15%										20%										25%										30%																																																																															
10000.										10200.										10400.										10600.										10800.										11000.										11200.																																																																															
35%										40%										45%										50%										55%										60%										65%																																																																															
11400.										11600.										11800.										12000.										12500.										13000.										13500.																																																																															
75%										75%										80%										85%										90%										95%										100%																																																																															
14000.										14667.										15333.										16000.										16667.										17333.										18000.																																																																															

191

DATA SHEET

PROBLEM

INCO - GUSS

PAGE

11

OF

17

DATE

10/20/73

WRITTEN

SE

TYPE	VARIABLE TITLE (Spread in 5 fields)					
1	PRODTWO		DIRECT		COST	PER UNIT
11.70	1.75	1.78	1.80	1.82	1.84	1.86
1.88	2.00	2.05	2.10	2.15	2.20	2.40
2.50	2.60	2.65	2.70	2.75	2.80	3.00
TYPE	VARIABLE TITLE (Spread in 5 fields)					
1	PRODTWO		MATURITY		VOLUME	UNITS
20000.	22500.	25000.	27500.	26000.	26500.	27000.
27500.	28000.	28500.	29000.	29500.	30000.	32500.
35000.	37500.	40000.	45000.	50000.	55000.	60000.

194

TYPE		VARIABLE TITLE (Spread in 5 fields)						
1. ADVERTISING EXPENSES PER YEAR		DOLLARS						
0%	5%	10%	15%	20%	25%	30%		
8000.	8100.	8200.	8300.	8400.	8500.	8600.		
35%	40%	45%	50%	55%	60%	65%		
8700.	8800.	8900.	9000.	9250.	9500.	9750.		
70%	75%	80%	85%	90%	95%	100%		
10000.	10250.	10500.	10750.	11000.	12000.	15000.		
2. ADVERTISING EXPENSES		START OF EXPENSE						
0%	5%	10%	15%	20%	25%	30%		
11.25	11.50	11.53	11.55	11.58	11.60	11.63		
35%	40%	45%	50%	55%	60%	65%		
11.65	11.68	11.70	11.73	11.75	11.80	11.85		
70%	75%	80%	85%	90%	95%	100%		
11.90	11.93	11.95	11.98	2.00	2.20	2.50		

TYPE										VARIABLE TITLE (Spread in 5 fields)																																																											
1										ADVERTISING EXPENSES										GROWTH PERIOD																																																	
1%										5%										10%										15%										20%										25%										30%									
2.75										2.83										2.92										3.00										3.10										3.20										3.30									
5%										10%										15%										20%										25%										30%																			
3.40										3.50										3.63										3.75										3.88										4.00										4.10									
75%										75%										80%										85%										90%										95%										100%									
4.20										4.30										4.40										4.50										5.00										5.50										6.00									
1%										5%										10%										15%										20%										25%										30%									
2000.										2100.										2200.										2300.										2400.										24250										24500.									
75%										75%										75%										75%										75%										75%										75%									
24750.										25000.										25250.										25500.										25750.										26000.										26500.									
75%										75%										75%										75%										75%										75%										75%									
27000.										27500.										28000.										30000.										32000.										34000.										36000.									

01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																										
TYPE										VARIABLE TITLE (Spread in 5 fields)																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																															
1										FIXED EXPENSES										START OF EXPENSE																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																					
										0%										5%										10%										15%										20%										25%										30%																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																			
0.50										0.75										1.00										1.03										1.05										1.08										1.10																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																													
										40%										45%										50%										55%										60%										65%																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																													
1.13										1.15										1.18										1.20										1.23										1.25										1.31																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																													
										75%										80%										85%										90%										95%										100%																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																													
1.38										1.44										1.50										1.75										2.00										2.50										3.00																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																													

01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
✓ COLUMN FILES																✓ FOOTNOTES																																																															
DESCRIPTION																																																																															
<p>NET PRESENT VALUE (N.P.V.) AT 5 PERCENT DISCOUNT RATE</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> <p>9</p> <p>10</p> <p>12</p> <p>15</p> <p>20</p> <p>25</p> </div> <div style="text-align: center;"> <p>9</p> <p>10</p> <p>12</p> <p>15</p> <p>20</p> <p>25</p> </div> </div>																																																																															
no footnotes																																																																															

DATA SHEET

PROBLEM

INCO - Guss

PAGE

17

OF

17

DATE

10/20/73

WRITTEN

SE

REPORT CONTROLS AND TITLE					TITLE OF REPORT				
MINBLK	MAXBLK	TITLE OF REPORT							
1	200	N. P. V. ANALYSIS							
REPORT SORTING AND FOOTNOTES CONTROLS									
NSORT	NFOOT	CENTER TITLE AROUND CENTERLINE							
7	0								
REPORTING SORTING CONTROLS									
KEY ROW	MINROW	MAXROW	MINCOL	MAXCOL	REMARKS NOT KEYPUNCHED				
1	1		1	200	MINBLK MAXBLK				
2	2		2	200					
3	3		3	200	MINROW				
4	4		4	200	KEYROW 2				
5	5		5	200	MAXROW				
6	6		6	200	LAP 1 MINCOL MAXCOL LAP 1' 4'				
7	7		7	200					
REMARKS NOT KEYPUNCHED									
FOOTNOTES 1 through 10									
FOOTNOTES 11 through 20									
FOOTNOTES 21 through 30									
FOOTNOTES 31 through 40									

B.11 The INCO Company Investment Problem--

GUSS Output

```

*****
*
*
*
*
*      THE INCO COMPANY  INVESTMENT PROBLEM
*
*
*
*****

```

INTEGER CONSTANTS & INDEXES

NUMBER OF INVESTMENT ITEMS	=	5
NUMBER OF PRODUCTS	=	2
NUMBER OF OVERHEAD EXPENSES	=	2
NUMBER OF INTEREST RATES	=	7
WORKING CAPITAL - OWN CODE	=	2
WORKING CAPITAL - NO DEPRECIATION CODE	=	0
LAND - LEASED CODE	=	1
LAND - NO DEPRECIATION CODE	=	0
BUILDINGS - OWNED CODE	=	3
SLM OF THE YEARS DIGITS DEPRECIATION CODE	=	3
HEAVY EQUIPMENT - OWNED CODE	=	3
DOUBLE RATE DECLINING BALANCE DEPRECIATION CODE	=	2
LIGHT EQUIPMENT - OWNED CODE	=	3
STRAIGHT LINE DEPRECIATION CODE	=	1

DETERMINISTIC VARIABLES

LIFE OF PROJECT	0.20000E 02
% DISCOUNT RATE # 1	C.50000E 01
% DISCOUNT RATE # 2	C.90000E 01
% DISCOUNT RATE # 3	0.10000E 02
% DISCOUNT RATE # 4	C.12000E 02
% DISCOUNT RATE # 5	0.15000E 02
% DISCOUNT RATE # 6	0.20000E 02
% DISCOUNT RATE # 7	0.25000E 02
WORKING CAPITAL -- TIME LAG FOR INVESTMENT	0.0
WORKING CAPITAL -- SERVICE LIFE	C.20000E 02
WORKING CAPITAL -- % SALVAGE AT END OF SERVICE	0.10000E 03
LAND -- TIME LAG FOR INVESTMENT	0.0
LAND -- SERVICE LIFE	C.20000E 02
LAND -- % SALVAGE AT END OF SERVICE	0.10000E 03
BUILDINGS -- TIME LAG FOR INVESTMENT	C.10000E 01
BUILDINGS -- SERVICE LIFE	0.25000E 02
BUILDINGS -- % SALVAGE AT END OF SERVICE	0.30000E 02
HEAVY EQUIPMENT -- TIME LAG FOR INVESTMENT	C.20000E 01
HEAVY EQUIPMENT -- SERVICE LIFE	0.10000E 02
HEAVY EQUIPMENT -- % SALVAGE AT END OF SERVICE	C.20000E 02
LIGHT EQUIPMENT -- TIME LAG FOR INVESTMENT	0.30000E 01
LIGHT EQUIPMENT -- SERVICE LIFE	0.50000E 01
LIGHT EQUIPMENT -- % SALVAGE AT END OF SERVICE	C.10000E 02

INPLT RANDOM VARIABLES

CLM.PCT	TAX	WORKING CAPITAL	LAND INVESTMENT	BUILDINGS INVESTMENT	HEAVY EQUIPMENT INVESTMENT	LIGHT EQUIPMENT INVESTMENT
0.	0.40000E 02	0.50000E 05	0.12000E 05	0.10000E 05	0.50000E 04	0.50000E 04
5.	0.41000E 02	0.51167E 05	0.12750E 05	0.10200E 05	0.10000E 05	0.10000E 05
10.	0.42000E 02	0.53333E 05	0.13500E 05	0.10400E 05	0.12500E 05	0.15000E 05
15.	0.43000E 02	0.55000E 05	0.14250E 05	0.10600E 05	0.15000E 05	0.20000E 05
20.	0.44000E 02	0.56667E 05	0.15000E 05	0.10800E 05	0.17500E 05	0.22500E 05
25.	0.45000E 02	0.58333E 05	0.15300E 05	0.11000E 05	0.20000E 05	0.25000E 05
30.	0.45500E 02	0.60000E 05	0.15600E 05	0.11200E 05	0.21000E 05	0.25417E 05
35.	0.46000E 02	0.61111E 05	0.15900E 05	0.11400E 05	0.22000E 05	0.25833E 05
40.	0.46500E 02	0.62222E 05	0.16200E 05	0.11600E 05	0.23000E 05	0.26250E 05
45.	0.47000E 02	0.63333E 05	0.16500E 05	0.11800E 05	0.24000E 05	0.26667E 05
50.	0.47500E 02	0.64444E 05	0.16800E 05	0.12000E 05	0.25000E 05	0.27083E 05
55.	0.48000E 02	0.65556E 05	0.17100E 05	0.12500E 05	0.26000E 05	0.27500E 05
60.	0.48500E 02	0.66667E 05	0.17400E 05	0.13000E 05	0.27000E 05	0.27917E 05
65.	0.49000E 02	0.67778E 05	0.17700E 05	0.13500E 05	0.28000E 05	0.28333E 05
70.	0.49500E 02	0.68889E 05	0.18000E 05	0.14000E 05	0.29000E 05	0.28750E 05
75.	0.50000E 02	0.70000E 05	0.18333E 05	0.14667E 05	0.30000E 05	0.29167E 05
80.	0.52500E 02	0.73333E 05	0.18667E 05	0.15333E 05	0.33333E 05	0.29583E 05
85.	0.55000E 02	0.76667E 05	0.19000E 05	0.16000E 05	0.36667E 05	0.30000E 05
90.	0.56700E 02	0.80000E 05	0.19333E 05	0.16667E 05	0.40000E 05	0.35000E 05
95.	0.58300E 02	0.85000E 05	0.19667E 05	0.17333E 05	0.45000E 05	0.40000E 05
100.	0.60000E 02	0.90000E 05	0.20000E 05	0.18000E 05	0.50000E 05	0.50000E 05
DIST. TYPE	1.	1.	1.	1.	1.	1.

0=DISCRETE, 1=CONTINUOUS

INPLT RANDCM VARIABLES

CUM.FCT	PRGDCNE PRICE PER UNIT	PRODCNE DIRECT COST PER UNIT	PRODCNE MATURITY VOLUME UNITS	PRDCNE START OF SELLING YEARS	PRGDCNE SALES GROWTH YEARS	PRODTWO PRICE PER UNIT
0.	0.22000E 01	0.15000E 01	0.50000E 05	0.15000E 01	0.50000E 00	0.25000E 01
5.	0.23500E 01	0.15700E 01	0.55000E 05	0.15300E 01	0.68000E 00	0.26800E 01
10.	0.25000E 01	0.16300E 01	0.60000E 05	0.15500E 01	0.75000E 00	0.27500E 01
15.	0.25500E 01	0.17000E 01	0.65000E 05	0.15800E 01	0.88000E 00	0.28100E 01
20.	0.26000E 01	0.17300E 01	0.70000E 05	0.16000E 01	0.10000E 01	0.28800E 01
25.	0.26500E 01	0.17500E 01	0.72500E 05	0.16300E 01	0.10600E 01	0.29400E 01
30.	0.27000E 01	0.17800E 01	0.75000E 05	0.16500E 01	0.11300E 01	0.30000E 01
35.	0.27400E 01	0.18000E 01	0.77500E 05	0.16800E 01	0.11880E 01	0.30300E 01
40.	0.27800E 01	0.18300E 01	0.80000E 05	0.17000E 01	0.12500E 01	0.30600E 01
45.	0.28100E 01	0.18500E 01	0.81000E 05	0.17300E 01	0.13130E 01	0.30900E 01
50.	0.28500E 01	0.18800E 01	0.82000E 05	0.17500E 01	0.13750E 01	0.31300E 01
55.	0.28900E 01	0.19000E 01	0.83000E 05	0.18000E 01	0.14380E 01	0.31600E 01
60.	0.29300E 01	0.19500E 01	0.84000E 05	0.18500E 01	0.15000E 01	0.31900E 01
65.	0.29600E 01	0.20000E 01	0.85000E 05	0.19000E 01	0.15800E 01	0.32200E 01
70.	0.30000E 01	0.20500E 01	0.86000E 05	0.19500E 01	0.16700E 01	0.32500E 01
75.	0.30500E 01	0.21000E 01	0.87000E 05	0.20000E 01	0.17500E 01	0.33100E 01
80.	0.31000E 01	0.21500E 01	0.88000E 05	0.21300E 01	0.18400E 01	0.33800E 01
85.	0.31500E 01	0.22000E 01	0.89000E 05	0.22500E 01	0.19200E 01	0.34400E 01
90.	0.32000E 01	0.22500E 01	0.90000E 05	0.23800E 01	0.20000E 01	0.35000E 01
95.	0.33500E 01	0.23000E 01	0.95000E 05	0.25000E 01	0.22500E 01	0.37500E 01
100.	0.35000E 01	0.25000E 01	0.10000E 06	0.27500E 01	0.25000E 01	0.40000E 01
DIST. TYPE	1.	1.	1.	1.	1.	1.

0=DISCRETE, 1=CONTINUOUS

INPUT RANCCM VARIABLES

CUM.PCT	PRCCTWO DIRECT COST PER UNIT	PRCCTWO MATURITY VOLUME UNITS	PRCCTWO START OF SELLING YEARS	PRCCTWO SALES GRWTH YEARS	ADVERTISING EXPENSES PER YEAR DOLLARS	ADVERTISING EXPENSES START OF EXPENSE
0.	0.17000E 01	0.20000E 05	0.22500E 01	0.22500E 01	0.80000E 04	0.12500E 01
5.	0.17500E 01	0.22500E 05	0.23000E 01	0.23000E 01	0.81000E 04	0.15000E 01
10.	0.17800E 01	0.25000E 05	0.23500E 01	0.23500E 01	0.82000E 04	0.15300E 01
15.	0.18000E 01	0.25500E 05	0.24000E 01	0.24000E 01	0.83000E 04	0.15500E 01
20.	0.18200E 01	0.26000E 05	0.25000E 01	0.24500E 01	0.84000E 04	0.15800E 01
25.	0.18400E 01	0.26500E 05	0.25500E 01	0.25000E 01	0.85000E 04	0.16000E 01
30.	0.18600E 01	0.27000E 05	0.26000E 01	0.25500E 01	0.86000E 04	0.16300E 01
35.	0.18800E 01	0.27500E 05	0.26500E 01	0.26000E 01	0.87000E 04	0.16500E 01
40.	0.20000E 01	0.28000E 05	0.27000E 01	0.26500E 01	0.88000E 04	0.16800E 01
45.	0.20500E 01	0.28500E 05	0.27500E 01	0.27000E 01	0.89000E 04	0.17000E 01
50.	0.21000E 01	0.29000E 05	0.28000E 01	0.27500E 01	0.90000E 04	0.17300E 01
55.	0.21500E 01	0.29500E 05	0.28500E 01	0.28000E 01	0.92500E 04	0.17500E 01
60.	0.22000E 01	0.30000E 05	0.29000E 01	0.28500E 01	0.95000E 04	0.18000E 01
65.	0.24000E 01	0.32500E 05	0.29500E 01	0.29000E 01	0.97500E 04	0.18500E 01
70.	0.25000E 01	0.35000E 05	0.30000E 01	0.29500E 01	0.10000E 05	0.19000E 01
75.	0.26000E 01	0.37500E 05	0.31000E 01	0.30000E 01	0.10250E 05	0.19300E 01
80.	0.26500E 01	0.40000E 05	0.32000E 01	0.30500E 01	0.10500E 05	0.19500E 01
85.	0.27000E 01	0.45000E 05	0.33000E 01	0.31000E 01	0.10750E 05	0.19800E 01
90.	0.27500E 01	0.50000E 05	0.34000E 01	0.31500E 01	0.11000E 05	0.20000E 01
95.	0.28000E 01	0.55000E 05	0.35000E 01	0.32000E 01	0.12000E 05	0.22000E 01
100.	0.30000E 01	0.60000E 05	0.37500E 01	0.32500E 01	0.15000E 05	0.25000E 01
DIST. TYPE						
	1.	1.	1.	1.	1.	1.

0=DISCRETE, 1=CONTINUOUS

INPUT RANDOM VARIABLES

CUM.PCT	ADVERTISING EXPENSES	FIXED EXPENSES PER YEAR	FIXED EXPENSES	FIXED EXPENSES
	GROWTH PERIOD	DOLLARS	START OF EXPENSE	GROWTH PERIOD
0.	0.27500E 01	0.20000E 05	0.50000E 00	0.10000E 01
5.	0.28300E 01	0.21000E 05	0.75000E 00	0.10500E 01
10.	0.29200E 01	0.22000E 05	0.10000E 01	0.11000E 01
15.	0.30000E 01	0.23000E 05	0.10300E 01	0.11500E 01
20.	0.31000E 01	0.24000E 05	0.10500E 01	0.12000E 01
25.	0.32000E 01	0.24250E 05	0.10800E 01	0.12500E 01
30.	0.33000E 01	0.24500E 05	0.11000E 01	0.13000E 01
35.	0.34000E 01	0.24750E 05	0.11300E 01	0.13500E 01
40.	0.35000E 01	0.25000E 05	0.11500E 01	0.14000E 01
45.	0.36300E 01	0.25250E 05	0.11800E 01	0.14500E 01
50.	0.37500E 01	0.25500E 05	0.12000E 01	0.15000E 01
55.	0.38800E 01	0.25750E 05	0.12300E 01	0.16000E 01
60.	0.40000E 01	0.26000E 05	0.12500E 01	0.17000E 01
65.	0.41000E 01	0.26500E 05	0.13100E 01	0.18000E 01
70.	0.42000E 01	0.27000E 05	0.13800E 01	0.19000E 01
75.	0.43000E 01	0.27500E 05	0.14400E 01	0.20000E 01
80.	0.44000E 01	0.28000E 05	0.15000E 01	0.22500E 01
85.	0.45000E 01	0.30000E 05	0.17500E 01	0.25000E 01
90.	0.50000E 01	0.32000E 05	0.20000E 01	0.30000E 01
95.	0.55000E 01	0.34000E 05	0.25000E 01	0.35000E 01
100.	0.60000E 01	0.36000E 05	0.30000E 01	0.40000E 01
DIST. TYPE	1.	1.	1.	1.

0=DISCRETE, 1=CONTINUOUS

CELLUM TITLES :

NET PRESENT VALUE (N.P.V.) AT 5 PERCENT DISCOUNT RATE
NET PRESENT VALUE (N.P.V.) AT 9 PERCENT DISCOUNT RATE
NET PRESENT VALUE (N.P.V.) AT 10 PERCENT DISCOUNT RATE
NET PRESENT VALUE (N.P.V.) AT 12 PERCENT DISCOUNT RATE
NET PRESENT VALUE (N.P.V.) AT 15 PERCENT DISCOUNT RATE
NET PRESENT VALUE (N.P.V.) AT 20 PERCENT DISCOUNT RATE
NET PRESENT VALUE (N.P.V.) AT 25 PERCENT DISCOUNT RATE

PCT.	1	2	3	4	5	6	7
100.0000	0.96471E C6	0.60081E 06	0.53550E 06	0.42637E 06	0.30333E 06	0.16837E 06	0.84973E 05
99.5000	0.64090E 06	0.39895E 06	0.36021E 06	0.29497E 06	0.22039E 06	0.13657E 06	0.82947E 05
99.0000	0.61229E 06	0.38980E 06	0.34501E 06	0.27394E 06	0.20286E 06	0.12366E 06	0.73375E 05
98.5000	0.60786E 06	0.37386E 06	0.33652E 06	0.27045E 06	0.18690E 06	0.11229E 06	0.65074E 05
98.0000	0.60770E 06	0.37215E 06	0.33003E 06	0.25980E 06	0.18606E 06	0.96232E 05	0.41675E 05
97.5000	0.58122E 06	0.35957E 06	0.31548E 06	0.25184E 06	0.18088E 06	0.54700E 05	0.41019E 05
97.0000	0.57810E 06	0.34692E 06	0.30555E 06	0.24216E 06	0.16026E 06	0.78572E 05	0.33434E 05
96.5000	0.55483E 06	0.34389E 06	0.30563E 06	0.23665E 06	0.15974E 06	0.77379E 05	0.31428E 05
96.0000	0.53744E 06	0.33511E 06	0.29590E 06	0.23057E 06	0.15726E 06	0.76088E 05	0.28399E 05
95.5000	0.53378E 06	0.31965E 06	0.28083E 06	0.21801E 06	0.15147E 06	0.71810E 05	0.25282E 05
95.0000	0.51110E 06	0.31262E 06	0.27716E 06	0.21622E 06	0.14390E 06	0.71303E 05	0.22509E 05
94.5000	0.50655E 06	0.29684E 06	0.25921E 06	0.19669E 06	0.13547E 06	0.65577E 05	0.18319E 05
94.0000	0.49858E 06	0.28621E 06	0.24593E 06	0.19386E 06	0.12810E 06	0.62915E 05	0.18094E 05
93.5000	0.49094E 06	0.28206E 06	0.24784E 06	0.19138E 06	0.12756E 06	0.59208E 05	0.16943E 05
93.0000	0.48584E 06	0.28198E 06	0.24735E 06	0.18976E 06	0.12683E 06	0.53304E 05	0.14611E 05
92.5000	0.48744E 06	0.28175E 06	0.24567E 06	0.18948E 06	0.12443E 06	0.52860E 05	0.94953E 04
92.0000	0.47649E 06	0.27867E 06	0.24561E 06	0.18619E 06	0.12284E 06	0.51316E 05	0.94407E 04
91.5000	0.47301E 06	0.27669E 06	0.24344E 06	0.18533E 06	0.11838E 06	0.51161E 05	0.85525E 04
91.0000	0.47214E 06	0.27494E 06	0.23810E 06	0.17550E 06	0.11730E 06	0.50672E 05	0.83779E 04
90.5000	0.45415E 06	0.27174E 06	0.23601E 06	0.17820E 06	0.11446E 06	0.47716E 05	0.62496E 04
90.0000	0.45388E 06	0.26916E 06	0.23550E 06	0.17693E 06	0.11435E 06	0.46849E 05	0.55381E 04
89.5000	0.45370E 06	0.26609E 06	0.23264E 06	0.17667E 06	0.11205E 06	0.46468E 05	0.48412E 04
89.0000	0.45298E 06	0.26485E 06	0.23238E 06	0.17364E 06	0.11116E 06	0.43851E 05	0.43444E 04
88.5000	0.44999E 06	0.26364E 06	0.22975E 06	0.17339E 06	0.11082E 06	0.43632E 05	0.35644E 04
88.0000	0.44785E 06	0.26344E 06	0.22568E 06	0.17139E 06	0.11641E 06	0.42238E 05	0.32393E 04
87.5000	0.44702E 06	0.25760E 06	0.22442E 06	0.17034E 06	0.11039E 06	0.40390E 05	0.16317E 04
87.0000	0.44254E 06	0.25631E 06	0.22385E 06	0.17025E 06	0.10532E 06	0.38867E 05	0.96224E 03
86.5000	0.43727E 06	0.25296E 06	0.22006E 06	0.16780E 06	0.10521E 06	0.38456E 05	0.38321E 04
86.0000	0.43545E 06	0.24585E 06	0.21766E 06	0.16423E 06	0.10463E 06	0.37465E 05	0.39700E 04
85.5000	0.43117E 06	0.24981E 06	0.21765E 06	0.16240E 06	0.10105E 06	0.37417E 05	0.43659E 04
85.0000	0.43101E 06	0.24788E 06	0.21538E 06	0.16139E 06	0.97980E 05	0.35621E 05	0.44439E 04
84.5000	0.41844E 06	0.24426E 06	0.21350E 06	0.15928E 06	0.95885E 05	0.31556E 05	0.53945E 04
84.0000	0.41741E 06	0.23947E 06	0.20709E 06	0.15409E 06	0.54835E 05	0.30709E 05	0.63131E 04
83.5000	0.41661E 06	0.23858E 06	0.20450E 06	0.14901E 06	0.94649E 05	0.26000E 05	0.82452E 04
83.0000	0.40910E 06	0.23170E 06	0.20013E 06	0.14755E 06	0.90807E 05	0.25086E 05	0.86337E 04
82.5000	0.35486E 06	0.22563E 06	0.19659E 06	0.14654E 06	0.88540E 05	0.24977E 05	0.96973E 04
82.0000	0.38525E 06	0.22523E 06	0.19569E 06	0.14604E 06	0.82226E 05	0.24637E 05	0.11319E 05
81.5000	0.38669E 06	0.21868E 06	0.18646E 06	0.13827E 06	0.82105E 05	0.24112E 05	0.14439E 05
81.0000	0.37738E 06	0.21308E 06	0.18398E 06	0.13575E 06	0.82020E 05	0.21565E 05	0.15342E 05
80.5000	0.36438E 06	0.20672E 06	0.17880E 06	0.13247E 06	0.83809E 05	0.21105E 05	0.15615E 05
80.0000	0.36425E 06	0.19491E 06	0.16615E 06	0.12189E 06	0.73886E 05	0.20263E 05	0.16061E 05
79.5000	0.36114E 06	0.19378E 06	0.16599E 06	0.11858E 06	0.72857E 05	0.16066E 05	0.18067E 05
79.0000	0.36095E 06	0.19262E 06	0.16381E 06	0.11847E 06	0.68581E 05	0.12579E 05	0.21297E 05
78.5000	0.35746E 06	0.19030E 06	0.16024E 06	0.11657E 06	0.65435E 05	0.11228E 05	0.22614E 05
78.0000	0.34335E 06	0.18893E 06	0.16016E 06	0.11432E 06	0.59660E 05	0.85873E 04	0.24326E 05
77.5000	0.33301E 06	0.18628E 06	0.15872E 06	0.11029E 06	0.59612E 05	0.54291E 04	0.25407E 05
77.0000	0.32736E 06	0.18233E 06	0.15840E 06	0.10882E 06	0.56616E 05	0.41097E 04	0.26607E 05
76.5000	0.32425E 06	0.17668E 06	0.15002E 06	0.10581E 06	0.55743E 05	0.38393E 04	0.27152E 05
76.0000	0.31700E 06	0.17429E 06	0.14787E 06	0.10451E 06	0.54958E 05	0.22212E 04	0.27769E 05
75.5000	0.31656E 06	0.16914E 06	0.14482E 06	0.10417E 06	0.52610E 05	0.17178E 04	0.29238E 05
75.0000	0.31600E 06	0.16781E 06	0.14150E 06	0.97932E 05	0.49567E 05	0.17996E 04	0.29348E 05
74.5000	0.30761E 06	0.16086E 06	0.13498E 06	0.92166E 05	0.49461E 05	0.21847E 04	0.29741E 05
74.0000	0.30752E 06	0.15839E 06	0.13084E 06	0.90207E 05	0.44730E 05	0.29874E 04	0.30241E 05
73.5000	0.30679E 06	0.15670E 06	0.13028E 06	0.86700E 05	0.42248E 05	0.41688E 04	0.30639E 05
73.0000	0.28590E 06	0.14926E 06	0.12671E 06	0.85500E 05	0.41897E 05	0.52216E 04	0.32405E 05
72.5000	0.28512E 06	0.14870E 06	0.12445E 06	0.84857E 05	0.40248E 05	0.52607E 04	0.33118E 05
72.0000	0.28150E 06	0.14679E 06	0.12342E 06	0.83334E 05	0.38648E 05	0.57369E 04	0.33831E 05
71.5000	0.27982E 06	0.13604E 06	0.11472E 06	0.79428E 05	0.37579E 05	0.70271E 04	0.35526E 05

71.0000	J.27760E 06	0.13435E 06	C.11C60E 06	0.78014E 05	0.36397E 05	-0.10487E 05	-0.37438E 05
70.5000	0.27277E 06	0.13392E 06	C.11C16E 06	0.75385E 05	C.35655E 05	-0.11442E 05	-0.37889E 05
70.0000	C.26644E 06	C.13192E 06	C.10819E 06	0.69975E 05	0.29119E 05	-0.11722E 05	-0.38126E 05
69.5000	0.26476E 06	0.13159E 06	C.10C63E 06	0.69685E 05	0.28946E 05	-0.12259E 05	-0.39436E 05
69.0000	0.26333E 06	J.13131E 06	0.10799E 06	C.69500E 05	C.26337E 05	-0.14713E 05	-0.39572E 05
68.5000	J.25785E 06	C.12959E 06	0.10710E 06	0.66203E 05	0.26212E 05	-0.15840E 05	-0.40268E 05
68.0000	J.25762E 06	0.12955E 06	C.10C59E 06	C.65900E 05	0.25998E 05	-0.15878E 05	-0.40636E 05
67.5000	0.25718E 06	0.12948E 06	J.10576E 06	0.65385E 05	0.22953E 05	-0.16581E 05	-0.41598E 05
67.0000	J.25336E 06	C.12913E 06	C.10538E 06	0.64759E 05	0.22428E 05	-0.18849E 05	-0.42742E 05
66.5000	J.24908E 06	0.12834E 06	J.10438E 06	C.63723E 05	0.22369E 05	-0.19190E 05	-0.42360E 05
66.0000	0.24677E 06	C.12093E 06	J.98984E 05	0.62703E 05	0.21468E 05	-0.20510E 05	-0.42995E 05
65.5000	J.24595E 06	0.12011E 06	C.57570E 05	0.62418E 05	0.21184E 05	-0.21166E 05	-0.44002E 05
65.0000	0.23828E 06	C.11620E 06	0.93015E 05	0.61026E 05	C.15617E 05	-0.21229E 05	-0.46949E 05
64.5000	J.22555E 06	C.11173E 06	0.92625E 05	0.60452E 05	0.19607E 05	-0.21710E 05	-0.47259E 05
64.0000	0.22472E 06	0.11022E 06	C.92245E 05	0.55674E 05	0.17602E 05	-0.22995E 05	-0.47333E 05
63.5000	C.22370E 06	0.10698E 06	J.86715E 05	0.54821E 05	0.17324E 05	-0.23316E 05	-0.48841E 05
63.0000	J.22032E 06	C.10546E 06	J.86173E 05	0.51930E 05	J.15657E 05	-0.23511E 05	-0.49101E 05
62.5000	J.21740E 06	J.99050E 05	0.78466E 05	C.50455E 05	0.14164E 05	-0.25690E 05	-0.49922E 05
62.0000	C.21198E 06	0.98799E 05	0.77102E 05	0.47909E 05	0.13830E 05	-0.28603E 05	-0.50265E 05
61.5000	0.21175E 06	0.95269E 05	C.77C18E 05	0.47382E 05	J.12795E 05	-0.29840E 05	-0.50462E 05
61.0000	J.20257E 06	0.94626E 05	C.77C14E 05	C.44663E 05	C.76352E 04	-0.30046E 05	-0.52181E 05
60.5000	C.15871E 06	0.91720E 05	J.76232E 05	0.41189E 05	0.62918E 04	-0.30664E 05	-0.53696E 05
60.0000	J.19489E 06	0.91072E 05	J.71499E 05	0.39213E 05	0.51570E 04	-0.31154E 05	-0.53728E 05
59.5000	J.19360E 06	0.86822E 05	0.68230E 05	0.37658E 05	C.40736E 04	-0.31341E 05	-0.54421E 05
59.0000	J.18878E 06	C.82435E 05	0.62727E 05	0.34618E 05	0.35377E 04	-0.31494E 05	-0.54467E 05
58.5000	J.18526E 06	J.75753E 05	C.61113E 05	C.31969E 05	0.34913E 04	-0.32262E 05	-0.54905E 05
58.0000	C.18374E 06	0.79570E 05	J.60516E 05	0.31266E 05	C.18282E 04	-0.34501E 05	-0.55205E 05
57.5000	J.18175E 06	C.79275E 05	J.58145E 05	0.29420E 05	-0.31773E 04	-0.35902E 05	-0.56079E 05
57.0000	J.18077E 06	0.74447E 05	C.56120E 05	C.29124E 05	-0.31473E 04	-0.37229E 05	-0.56219E 05
56.5000	C.18064E 06	0.72598E 05	0.55167E 05	0.27538E 05	-0.34578E 04	-0.38551E 05	-0.57270E 05
56.0000	J.17507E 06	0.72239E 05	0.54212E 05	0.25440E 05	-0.35015E 04	-0.39361E 05	-0.57376E 05
55.5000	J.17696E 06	0.69224E 05	C.53436E 05	C.23299E 05	-0.37711E 04	-0.39797E 05	-0.58557E 05
55.0000	0.17610E 06	C.66174E 05	0.51219E 05	0.22617E 05	-0.37015E 05	-0.40124E 05	-0.58621E 05
54.5000	J.17311E 06	0.64425E 05	J.45708E 05	0.20425E 05	-0.30997E 05	-0.40627E 05	-0.60820E 05
54.0000	0.16649E 06	0.64396E 05	0.47667E 05	0.18739E 05	-0.31343E 05	-0.42355E 05	-0.62747E 05
53.5000	0.16223E 06	C.64C20E 05	0.46121E 05	0.17360E 05	-0.31566E 05	-0.43060E 05	-0.63895E 05
53.0000	J.15203E 06	0.63569E 05	0.44315E 05	C.14947E 05	-0.31638E 05	-0.45875E 05	-0.64392E 05
52.5000	C.14993E 06	0.62291E 05	0.43730E 05	0.12484E 05	-0.31693E 05	-0.46624E 05	-0.65193E 05
52.0000	0.14960E 06	C.61778E 05	0.42669E 05	0.10591E 05	-0.31785E 05	-0.47592E 05	-0.65773E 05
51.5000	0.14877E 06	0.57381E 05	0.39212E 05	C.94452E 04	-0.32289E 05	-0.49814E 05	-0.67795E 05
51.0000	0.14406E 06	0.51000E 05	0.36375E 05	0.86253E 04	-0.32292E 05	-0.45934E 05	-0.68441E 05
50.5000	0.14308E 06	0.47459E 05	C.25789E 05	0.57636E 04	-0.23416E 05	-0.51218E 05	-0.68463E 05
50.0000	0.14279E 06	0.43202E 05	C.26559E 05	J.28906E 04	-0.23937E 05	-0.53967E 05	-0.68562E 05
49.5000	C.12968E 06	C.42073E 05	J.25967E 05	0.89427E 03	-0.25338E 05	-0.55662E 05	-0.69034E 05
49.0000	0.13954E 06	0.40953E 05	C.25350E 05	0.18904E 03	-0.26726E 05	-0.56138E 05	-0.69095E 05
48.5000	0.13582E 06	0.39302E 05	J.24734E 05	-0.41593E 02	-0.27655E 05	-0.56589E 05	-0.69491E 05
48.0000	0.13123E 06	C.38283E 05	J.21422E 05	-0.66329E 03	-0.28009E 05	-0.57264E 05	-0.69557E 05
47.5000	0.12484E 06	0.33902E 05	J.21296E 05	-0.30650E 04	-0.28077E 05	-0.57366E 05	-0.70688E 05
47.0000	0.11913E 06	0.33807E 05	0.21149E 05	-0.31493E 04	-0.30287E 05	-0.57392E 05	-0.71795E 05
46.5000	J.11895E 06	0.33544E 05	0.19631E 05	-0.35341E 04	-0.30538E 05	-0.57536E 05	-0.72289E 05
46.0000	0.11616E 06	0.32594E 05	0.18682E 05	-0.53490E 04	-0.30742E 05	-0.58074E 05	-0.72602E 05
45.5000	0.11411E 06	0.32063E 05	0.17389E 05	-0.57903E 04	-0.32589E 05	-0.55084E 05	-0.73713E 05
45.0000	0.11391E 06	0.31575E 05	0.15338E 05	-0.80908E 04	-0.33711E 05	-0.60973E 05	-0.75571E 05
44.5000	J.11314E 06	0.29509E 05	0.14098E 05	-0.84083E 04	-0.34142E 05	-0.61017E 05	-0.75692E 05
44.0000	0.10950E 06	C.25050E 05	C.13621E 05	-0.10625E 05	-0.35224E 05	-0.62282E 05	-0.77033E 05
43.5000	0.10902E 06	0.28814E 05	0.13622E 05	-0.11032E 05	-0.36684E 05	-0.62710E 05	-0.77431E 05
43.0000	0.10844E 06	0.27618E 05	0.13641E 05	-0.11091E 05	-0.36713E 05	-0.63138E 05	-0.77637E 05
42.5000	0.10756E 06	C.27463E 05	0.12606E 05	-0.11428E 05	-0.37090E 05	-0.64297E 05	-0.78073E 05
42.0000	0.10755E 06	0.27145E 05	J.10605E 05	-0.12736E 05	-0.38440E 05	-0.65238E 05	-0.78643E 05
41.5000	0.106470E 06	0.20940E 05	J.10497E 05	-0.13251E 05	-0.38473E 05	-0.65339E 05	-0.79691E 05
41.0000	0.10311E 06	0.15902E 05	C.72580E 04	-0.14494E 05	-0.38486E 05	-0.65571E 05	-0.79773E 05
40.5000	0.10149E 06	0.19665E 05	0.65144E 04	-0.16515E 05	-0.38603E 05	-0.65626E 05	-0.81100E 05
40.0000	0.97113E 05	0.18873E 05	C.46C21E 04	-0.16555E 05	-0.39724E 05	-0.65950E 05	-0.81271E 05
39.5000	0.96124E 05	0.18292E 05	C.45843E 04	-0.16602E 05	-0.35942E 05	-0.66069E 05	-0.81290E 05
39.0000	J.94163E 05	0.16045E 05	J.36317E 04	-0.18543E 05	-0.42149E 05	-0.66853E 05	-0.81862E 05
38.5000	0.93457E 05	0.14747E 05	C.21C73E 04	-0.18666E 05	-0.43224E 05	-0.67493E 05	-0.82707E 05

38.CCCC	0.92510E C5	0.14712E C5	0.13750E C4	-0.19762E C5	-0.43278E C5	-0.67918F C5	-0.83129E C5
37.5000	0.88525E C5	0.13570E C5	-0.22212E C3	-0.20257E C5	-0.45464E C5	-0.69264E C5	-0.83134E C5
37.0000	0.86843E C5	0.11643E C5	-0.65932E C3	-0.21298E C5	-0.45702E C5	-0.71455E C5	-0.83347E C5
36.5000	0.85479E C5	0.11104E C5	-0.92000E C3	-0.21787E C5	-0.46158E C5	-0.71758E C5	-0.84490E C5
36.0000	0.83586E C5	0.11095E C5	-0.33253E C4	-0.25635E C5	-0.49195E C5	-0.72680E C5	-0.85131E C5
35.5000	0.79792E C5	0.10615E C5	-0.35257E C4	-0.25984E C5	-0.50314E C5	-0.72756E C5	-0.85268E C5
35.CCCC	0.77247E C5	0.51657E C4	-0.51375E C4	-0.27233E C5	-0.52464E C5	-0.73977E C5	-0.85861E C5
34.5000	0.65223E C5	0.50350E C4	-0.11119E C5	-0.31133E C5	-0.52570E C5	-0.74258E C5	-0.86416E C5
34.CCCC	0.62243E C5	-0.39081E C4	-0.14293E C5	-0.35484E C5	-0.54085E C5	-0.74639E C5	-0.88422E C5
33.5000	0.60760E C5	-0.72853E C4	-0.18053E C5	-0.36641E C5	-0.55468E C5	-0.75011E C5	-0.88749E C5
33.0000	0.57506E C5	-0.79177E C4	-0.18514E C5	-0.37003E C5	-0.56838E C5	-0.76502E C5	-0.89133F C5
32.5000	0.57243E C5	-0.13848E C5	-0.25693E C5	-0.41822E C5	-0.57385E C5	-0.77394E C5	-0.89848E C5
32.0000	0.56495E C5	-0.13645E C5	-0.25722E C5	-0.42186E C5	-0.59549E C5	-0.77945E C5	-0.90615E C5
31.5000	0.56179E C5	-0.15487E C5	-0.25656E C5	-0.42479E C5	-0.60287E C5	-0.78572E C5	-0.91145E C5
31.CCCC	0.49210F C5	-0.15653E C5	-0.26564E C5	-0.44808E C5	-0.60555E C5	-0.79442E C5	-0.91397E C5
30.5000	0.48474E C5	-0.17854E C5	-0.27708E C5	-0.45264E C5	-0.64034E C5	-0.79599E C5	-0.91662F C5
30.CCCC	0.45989E C5	-0.18650E C5	-0.29771E C5	-0.46741E C5	-0.64684E C5	-0.82440E C5	-0.91971E C5
29.5000	0.45396E C5	-0.18555E C5	-0.30298E C5	-0.46904E C5	-0.65143E C5	-0.85482E C5	-0.92473E C5
29.0000	0.45084E C5	-0.19220F C5	-0.31355E C5	-0.46948E C5	-0.66452E C5	-0.85899E C5	-0.93816E C5
28.5000	0.43795F C5	-0.20230E C5	-0.31720E C5	-0.47226E C5	-0.66190E C5	-0.87083F C5	-0.94065E C5
28.0000	0.39231F C5	-0.23418E C5	-0.33713E C5	-0.50141E C5	-0.68638E C5	-0.87249E C5	-0.94425E C5
27.5000	0.32956E C5	-0.25875E C5	-0.32547E C5	-0.53130E C5	-0.69439E C5	-0.88014F C5	-0.94507E C5
27.CCCC	0.31341E C5	-0.26496E C5	-0.35083E C5	-0.53252E C5	-0.73145E C5	-0.88672E C5	-0.96586E C5
26.5000	0.23852E C5	-0.26694E C5	-0.37627E C5	-0.53890E C5	-0.73861E C5	-0.89410E C5	-0.96559E C5
26.CCCC	0.22513E C5	-0.32859E C5	-0.40657E C5	-0.55062E C5	-0.75391E C5	-0.89600E C5	-0.97138F C5
25.5000	0.21221E C5	-0.35659E C5	-0.45040E C5	-0.59964E C5	-0.76230E C5	-0.89791F C5	-0.97345F C5
25.0000	0.17067E C5	-0.36729E C5	-0.48654E C5	-0.64634E C5	-0.76981E C5	-0.89911E C5	-0.97571E C5
24.5000	0.14352E C5	-0.38314E C5	-0.48642E C5	-0.65171E C5	-0.77133E C5	-0.90236E C5	-0.98569E C5
24.CCCC	0.98248E C4	-0.45206E C5	-0.52705E C5	-0.66159E C5	-0.77987E C5	-0.90258E C5	-0.10190E C6
23.5000	0.35512E C5	-0.46631E C5	-0.54236E C5	-0.66370E C5	-0.79023E C5	-0.91254F C5	-0.10243E C6
23.CCCC	-0.52792E C3	-0.48892E C5	-0.57851E C5	-0.67470E C5	-0.80406E C5	-0.93780E C5	-0.10438E C6
22.5000	-0.55885E C4	-0.51457E C5	-0.57941E C5	-0.67878E C5	-0.82073E C5	-0.99094E C5	-0.10461F C6
22.0000	-0.11703F C5	-0.51888E C5	-0.58525E C5	-0.70571E C5	-0.82489E C5	-0.99325E C5	-0.10809E C6
21.5000	-0.14530E C5	-0.54910E C5	-0.60977E C5	-0.73709E C5	-0.85021E C5	-0.10184E C6	-0.11081E C6
21.CCCC	-0.15377E C5	-0.59985E C5	-0.65343E C5	-0.73797E C5	-0.89472E C5	-0.10298E C6	-0.11098E C6
20.5000	-0.17624E C5	-0.60163E C5	-0.69008E C5	-0.83006E C5	-0.94049E C5	-0.10400E C6	-0.11123E C6
20.CCCC	-0.25417E C5	-0.67157E C5	-0.74224E C5	-0.84333E C5	-0.97351E C5	-0.10952E C6	-0.11181E C6
19.5000	-0.26547E C5	-0.67635E C5	-0.75475E C5	-0.88488E C5	-0.10150E C6	-0.11068E C6	-0.11508F C6
19.0000	-0.27402E C5	-0.72323E C5	-0.79606E C5	-0.91114E C5	-0.10292E C6	-0.11293E C6	-0.11703F C6
18.5000	-0.30511E C5	-0.77230E C5	-0.84647E C5	-0.96220E C5	-0.10528E C6	-0.11406E C6	-0.11707F C6
18.0000	-0.37805F C5	-0.811174E C5	-0.87585E C5	-0.98473E C5	-0.10773E C6	-0.11684E C6	-0.11768E C6
17.5000	-0.56622E C5	-0.87413E C5	-0.91982E C5	-0.98879E C5	-0.10862E C6	-0.11767E C6	-0.11776E C6
17.0000	-0.57444E C5	-0.93292E C5	-0.98496E C5	-0.10639E C6	-0.11378E C6	-0.11767E C6	-0.11819E C6
16.5000	-0.59263E C5	-0.97164E C5	-0.10266E C6	-0.11183E C6	-0.11806E C6	-0.11900E C6	-0.11955E C6
16.CCCC	-0.59834E C5	-0.10117E C6	-0.10813E C6	-0.11508E C6	-0.11910E C6	-0.11934F C6	-0.11963E C6
15.5000	-0.88263E C5	-0.10842F C6	-0.11119E C6	-0.11844E C6	-0.12035E C6	-0.12167E C6	-0.12072F C6
15.CCCC	-0.93494E C5	-0.11435E C6	-0.11670E C6	-0.11883E C6	-0.12272E C6	-0.12304E C6	-0.12106E C6
14.5000	-0.95242E C5	-0.11524E C6	-0.11688E C6	-0.12030E C6	-0.12448E C6	-0.12455E C6	-0.12146E C6
14.0000	-0.10272E C6	-0.12038E C6	-0.12347E C6	-0.12506E C6	-0.12763E C6	-0.12484E C6	-0.12161E C6
13.5000	-0.10338E C6	-0.12117E C6	-0.12411E C6	-0.12652E C6	-0.12849E C6	-0.12720E C6	-0.12329E C6
13.0000	-0.10346E C6	-0.12370E C6	-0.12446E C6	-0.12786E C6	-0.12919E C6	-0.12790E C6	-0.12542E C6
12.5000	-0.10601E C6	-0.12558E C6	-0.12674E C6	-0.12935E C6	-0.13146E C6	-0.12822E C6	-0.12614E C6
12.CCCC	-0.10720E C6	-0.12559E C6	-0.12797E C6	-0.13112E C6	-0.13232E C6	-0.12879E C6	-0.12706E C6
11.5000	-0.11420E C6	-0.12795E C6	-0.12955E C6	-0.13148E C6	-0.13314E C6	-0.13109E C6	-0.12862E C6
11.0000	-0.11439E C6	-0.12904E C6	-0.13106E C6	-0.13187E C6	-0.13317E C6	-0.13290F C6	-0.12872E C6
10.5000	-0.11522E C6	-0.13014E C6	-0.13218E C6	-0.13504E C6	-0.13338E C6	-0.13367E C6	-0.12945E C6
10.0000	-0.11883E C6	-0.13182E C6	-0.13230E C6	-0.13630E C6	-0.13444E C6	-0.13386E C6	-0.13046E C6
9.5000	-0.12036E C6	-0.13474E C6	-0.13213E C6	-0.13654E C6	-0.13559E C6	-0.13450F C6	-0.13077E C6
9.CCCC	-0.12337E C6	-0.13855E C6	-0.13932E C6	-0.14184E C6	-0.13980E C6	-0.13454E C6	-0.13094E C6
8.5000	-0.12587E C6	-0.13887E C6	-0.14035E C6	-0.14325E C6	-0.14032E C6	-0.13749E C6	-0.13244E C6
8.0000	-0.13486E C6	-0.14146E C6	-0.14333E C6	-0.14310E C6	-0.14123E C6	-0.13767E C6	-0.13262E C6
7.5000	-0.13915E C6	-0.14428E C6	-0.14415E C6	-0.14543E C6	-0.14274E C6	-0.13844E C6	-0.13760E C6
7.0000	-0.13591E C6	-0.14685E C6	-0.14660E C6	-0.14562E C6	-0.14340E C6	-0.14060E C6	-0.13824E C6
6.5000	-0.14358E C6	-0.15004E C6	-0.14744E C6	-0.14550E C6	-0.14670E C6	-0.14161E C6	-0.13889E C6
6.0000	-0.15092E C6	-0.15048E C6	-0.14878E C6	-0.14604E C6	-0.14868E C6	-0.14412E C6	-0.13969E C6
5.5000	-0.15232E C6	-0.15391E C6	-0.15567E C6	-0.15463E C6	-0.15175E C6	-0.14540E C6	-0.14064E C6

5.0000	-0.16365E 06	-0.15576E 06	-0.15622E 06	-0.15604E 06	-0.15180E 06	-0.14579E 06	-0.14267E 06
4.5000	-0.16430E 06	-0.16113E 06	-0.15972E 06	-0.15662E 06	-0.15191E 06	-0.15275E 06	-0.14522E 06
4.0000	-0.16544E 06	-0.16416E 06	-0.16139E 06	-0.15909E 06	-0.15538E 06	-0.15543E 06	-0.14671E 06
3.5000	-0.17492E 06	-0.16604E 06	-0.16527E 06	-0.16318E 06	-0.16051E 06	-0.15754E 06	-0.14935E 06
3.0000	-0.19705E 06	-0.18757E 06	-0.18481E 06	-0.17925E 06	-0.17118E 06	-0.15600E 06	-0.15531E 06
2.5000	-0.21899E 06	-0.20339E 06	-0.19171E 06	-0.18741E 06	-0.17434E 06	-0.15924E 06	-0.15563E 06
2.0000	-0.23003E 06	-0.20628E 06	-0.20290E 06	-0.19622E 06	-0.18678E 06	-0.17270E 06	-0.15826E 06
1.5000	-0.26386E 06	-0.22907E 06	-0.22187E 06	-0.20897E 06	-0.19289E 06	-0.17308E 06	-0.16187E 06
1.0000	-0.30015E 06	-0.25434E 06	-0.24515E 06	-0.22893E 06	-0.20903E 06	-0.18449E 06	-0.16718E 06
0.5000	-0.41321E 06	-0.34036E 06	-0.32603E 06	-0.30094E 06	-0.27053E 06	-0.23355E 06	-0.20778E 06

SUMMARY STATISTICS

DISTRIBUTION SUMMARY

PCT.	1	2	3	4	5	6	7
100.0000	0.96471E 06	0.60081E 06	0.53550E 06	0.42637E 06	0.30333E 06	0.16837E 06	0.84973E 05
90.0000	0.45388E 06	0.26916E 06	0.23550E 06	0.17693E 06	0.11435E 06	0.46849E 05	0.55381E 04
80.0000	0.36425E 06	0.19491E 06	0.16615E 06	0.12189E 06	0.73886E 05	0.20263E 05	-0.16061E 05
70.0000	0.26644E 06	0.13192E 06	0.10619E 06	0.69975E 05	0.29119E 05	-0.11722E 05	-0.38126E 05
60.0000	0.19489E 06	0.91072E 05	0.71499E 05	0.39213E 05	0.51570E 04	-0.31154E 05	-0.53728E 05
50.0000	0.14279E 06	0.43202E 05	0.26559E 05	0.28906E 04	-0.23937E 05	-0.53967E 05	-0.68562E 05
40.0000	0.97113E 05	0.18873E 05	0.46621E 04	-0.16555E 05	-0.39724E 05	-0.65950E 05	-0.81271E 05
30.0000	0.45989E 05	-0.18850E 05	-0.29771E 05	-0.46741E 05	-0.64684E 05	-0.82440E 05	-0.91971E 05
20.0000	-0.25417E 05	-0.67157E 05	-0.74224E 05	-0.84333E 05	-0.97351E 05	-0.10992E 06	-0.11181E 06
10.0000	-0.11883E 06	-0.13182E 06	-0.13330E 06	-0.13630E 06	-0.13444E 06	-0.13386E 06	-0.13046E 06
0.5000	-0.41321E 06	-0.34036E 06	-0.32603E 06	-0.30094E 06	-0.27053E 06	-0.23355E 06	-0.20778E 06

DISTRIBUTION PARAMETERS

MEAN	0.16513E 06	0.64860E 05	0.47477E 05	0.18963E 05	-0.12198E 05	-0.44731E 05	-0.63557E 05
ST. DEV.	0.21874E 06	0.15138E 06	0.13914E 06	0.11854E 06	0.95090E 05	0.68982E 05	0.52560E 05

B.12 The INCO Company Investment Problem--

PRODIP Input

01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
1ST CARD																																																																															
PROBLEM IDENTIFICATION TITLE																																																																															
THE INCO COMPANY INVESTMENT PROBLEM																																																																															
CENTER TITLE AROUND CENTERLINE																																																																															
2ND CARD																																																																															
ENTER: 1 = Desired 0 = Skip																																																																															
PEARSON'S OPTION A:																																																																															
1																																																																															
3RD CARD																																																																															
ENTER: 1 = Desired 0 = Skip																																																																															
CONTINUE OPTION B:																																																																															
EXPONENTIAL NORMAL LOGNORMAL FISHER-TIPPETT I FISHER-TIPPETT II WEIBULL BETA																																																																															
1 1 1 1 1 1 1 1																																																																															
4TH CARD																																																																															
ENTER: 1 = Desired 0 = Skip																																																																															
DISCRETE OPTION C:																																																																															
POISSON TRUNCATED POISSON BINOMIAL NEGATIVE BINOMIAL TRUNCATED NEG. BINOMIAL																																																																															
1 0 1 1 1 1																																																																															
5TH CARD																																																																															
ENTER: 1 = Desired 0 = Skip																																																																															
N NV H SMF DRY P LR																																																																															
200 10 0 5 0 0 0																																																																															

216

B.13 The INCO Company Investment Problem--

PRODIP Output

```
*****
*
*
*
*      THE INCO COMPANY  INVESTMENT PROBLEM
*
*
*
*****
```


THE INCO COMPANY INVESTMENT PROBLEM
PEARSON SYSTEM

DISTRIBUTION TYPE 3

K = -0.406949060 00
B1 = 0.957111890-01
B2 = 0.305323930 01
LAMBDA= 0.687055260 03
H = 0.0
N = 200

PM1 = 0.550125000 01	PM2 = 0.325515630 02	PM3 = 0.205316480 03	PM4 = 0.137085970 04	PM5 = 0.962916470 04
PM6 = 0.707231110 05	PM7 = 0.535818890 06	PM8 = 0.425644940 07	M1 = 0.550125000 01	M2 = 0.228781090 01
S = 0.151255110 01	M3 = 0.107056130 01	M4 = 0.159808950 02	M5 = 0.180914160 02	M6 = 0.157293480 03
M8 = 0.172997750 04	BE1 = 0.957111890-01	BE2 = 0.305323930 01	BE3 = 0.706974240 00	BE4 = 0.131356130 02
BE6 = 0.63140470 02	SIGB1= 0.781462630-01	SIGB2= 0.197545450 00	SIGL= 0.335165650 04	SIGK= 0.909206620 00
A = 0.119258240 02	B = 0.113838590 02	C = 0.605323930 01	D = 0.115645140 02	E = 0.180655000 00
F = -0.180655000 00	C1 = 0.565192930 01	C2 = -0.180655000 00	SGC1= 0.109336700 01	SGC2= 0.459410890 00

XI

0.250000000 01
0.375000000 01
0.425000000 01
0.475000000 01
0.525000000 01
0.575000000 01
0.625000000 01
0.675000000 01
0.725000000 01
0.900000000 01

FI

0.110000000 02
0.220000000 02
0.150000000 02
0.270000000 02
0.340000000 02
0.230000000 02
0.200000000 02
0.130000000 02
0.230000000 02
0.120000000 02

SAMPLE STATISTICS

XB	XT	XI	FI	CFI	DFC
0.150000D 01	0.350000D 01	0.250000D 01	0.110000D 02	0.110000D 02	0.550000D-01
0.350000D 01	0.400000D 01	0.375000D 01	0.220000D 02	0.330000D 02	0.165000D 00
0.400000D 01	0.450000D 01	0.425000D 01	0.150000D 02	0.480000D 02	0.240000D 00
0.450000D 01	0.500000D 01	0.475000D 01	0.270000D 02	0.750000D 02	0.375000D 00
0.500000D 01	0.550000D 01	0.525000D 01	0.340000D 02	0.109000D 03	0.545000D 00
0.550000D 01	0.600000D 01	0.575000D 01	0.230000D 02	0.132000D 03	0.660000D 00
0.600000D 01	0.650000D 01	0.625000D 01	0.200000D 02	0.152000D 03	0.760000D 00
0.650000D 01	0.700000D 01	0.675000D 01	0.130000D 02	0.165000D 03	0.825000D 00
0.700000D 01	0.750000D 01	0.725000D 01	0.230000D 02	0.188000D 03	0.940000D 00
0.750000D 01	0.105000D 02	0.900000D 01	0.120000D 02	0.200000D 03	0.100000D 01

THE INCO COMPANY INVESTMENT PROBLEM
EXPONENTIAL DISTRIBUTION

DENSITY FUNCTION $G(X) = \text{LAMBDA1} \exp(-\text{LAMBDA1} X)$, $X=0$, $\text{LAMBDA1}=0$

DISTRIBUTION FUNCTION $F(X) = 1 - \exp(-\text{LAMBDA1} X)$

$\text{LAMBDA1} = 1/M1$

H(X)	FE
0.470710130 00	0.941420260 02
0.516695060 00	0.919698540 01
0.558684800 00	0.839794750 01
0.597026450 00	0.766833020 01
0.632036960 00	0.700210250 01
0.664005740 00	0.639375690 01
0.693197070 00	0.583826460 01
0.719852230 00	0.533103370 01
0.744191590 00	0.486787120 01
0.851720630 00	0.215058070 02

MAX ABS DIFF (CFO-H(X))= 0.415710130 00
 $\text{LAMBDA1} = 0.181776870 00$
 $\text{CHISQ} = 0.340606680 03$
 $\text{NG}(\text{CHISQ}) = 9$ $\text{SMFE} = 0.500000000 01$
 $\text{P}(\text{EXCEEDING CHISQ}) = \text{NO TEST}$ $\text{DF} = 7$

THE INCO COMPANY INVESTMENT PROBLEM

NORMAL DISTRIBUTION

DENSITY FUNCTION $G(X)=1/S*\text{SQRT}(2\text{PI}) \text{ EXP}(-.5(X-M/S)**2),$

$-\text{INFINITY}+X+\text{INFINITY}, -\text{INFINITY}+M+\text{INFINITY}, S=0$

DISTRIBUTION FUNCTION $H(X)=1/S*\text{SQRT}(2\text{PI})*\text{INTEGRAL}(\text{EXP}(-(T-M)**2/2*S**2)DT)$

M=MEAN, S=STANDARD DEVIATION

Z	H(X)	FE
-0.13230958D 01	0.92901641D-01	0.18580328D 02
-0.99252843D 00	0.16046998D 00	0.13513668D 02
-0.66196109D 00	0.25359814D 00	0.18705631D 02
-0.33139375D 00	0.37017343D 00	0.23235058D 02
-0.82641834D-03	0.49967031D 00	0.25899377D 02
0.32974092D 00	0.62920225D 00	0.25906387D 02
0.66030825D 00	0.74547193D 00	0.23253936D 02
0.99087559D 00	0.83912676D 00	0.18730967D 02
0.13214429D 01	0.90682327D 00	0.13539301D 02
0.33048469D 01	0.99952482D 00	0.18540312D 02

MAX ABS DIFF (DFG-H(X))= 0.45329690D-01

M= 0.55012500D 01 S= 0.15125511D 01

CHISQ= 0.23752422D 02

NG(CHISQ)=10 SMFE= 0.50000000D 01

P(EXCEEDING CHISQ)= 0.12593299D-02

CF= 7

THE INCO COMPANY INVESTMENT PROBLEM
SAMPLE STATISTICS LCGNORMAL DISTRIBUTION

YT	YI	FI	CFI	DFD
0.125276D 01	C.916291D 00	0.110000D 02	0.110000D 02	0.550000D-01
0.138629D 01	0.132176D 01	0.220000D 02	0.330000D 02	0.165000D 00
C.150400D 01	C.144692D 01	0.150000D 02	0.480000D 02	0.240000D 00
0.160944D 01	0.155814D 01	C.270000D 02	0.750000D 02	0.375000D 00
0.170475D 01	0.165823D 01	0.340000D 02	0.109000D 03	C.545000D 00
0.179176D 01	C.174920D 01	0.230000D 02	0.132000D 03	0.660000D 00
0.187180D 01	0.183258D 01	C.200000D 02	0.152000D 03	0.760000D 00
0.194591D 01	C.190954D 01	0.130000D 02	0.165000D 03	C.825000D 00
0.201490D 01	C.198100D 01	0.230000D 02	0.188000D 03	0.940000D 00
0.235138D 01	0.219722D 01	0.120000D 02	0.200000D 03	0.100000D 01

THE INCO COMPANY INVESTMENT PROBLEM
LOGNORMAL DISTRIBUTION

DENSITY FUNCTION $G(Y) = 1/S \cdot \text{SQRT}(2\text{PI}) \cdot \text{EXP}(-.5(Y-M/S)**2)$,

$-INFINITY+Y+INFINITY, -INFINITY+M+INFINITY, S=0$

DISTRIBUTION FUNCTION $H(X) = 1/S \cdot \text{SQRT}(2\text{PI}) \cdot \text{INTEGRAL}(\text{EXP}(-(T-M)**2/2*S**2)DT)$

$YI = \text{LN}(XI), YB = \text{LN}(XB), YT = \text{LN}(XT)$

$M = \text{MEAN}(YI), S = \text{STANDARD DEVIATION}(YI)$

Z	H(X)	FE
-0.141393C4D 01	0.78691039D-01	0.15738208D 02
-0.95564404D 00	0.16962614D 00	0.18187021D 02
-0.55140681C 00	0.29067738D 00	0.2421C246D 02
-0.18980428D 00	0.42473129D 00	0.26810783D 02
0.137305CCD 00	0.55460504D 00	0.25974750D 02
0.43593237C 00	0.66855722D 00	0.2279C437D 02
0.71064292D 00	0.76134714D 00	0.18557983D 02
0.96498517D 00	0.83272379D 00	0.14275331D 02
0.12017721C 01	0.88527416D 00	0.1051C075D 02
0.23565616D 01	0.99077749D 00	0.21100665D 02

MAX ABS DIFF (DFC-H(X))= 0.54725835D-01

M= 0.16647414D 01 S= 0.291371C7D 00

CHISQ= 0.272C6327C 02

NG(CHISQ)=1C SMFE= 0.50000000D 01

P(EXCEEDING CHISQ)= 0.30596103D-03

M2= 0.84897100D-01

H(Y)= 0.0

DF= 7

THE INCC COMPANY INVESTMENT PROBLEM

FISHER-TIPPETT TYPE 1 DISTRIBUTION

DENSITY FUNCTION $G(X) = \text{ALPHA} \exp(-Y - \exp(-Y))$ WHERE

$Y = \text{ALPHA}(X - U)$, $-\infty < X < \infty$, $\text{ALPHA} > 0$, $-\infty < U < \infty$

DISTRIBUTION FUNCTION $H(X) = \exp(-\exp(-Y))$

$U = \text{MODE}$, $\text{ALPHA} = \text{SCALE PARAMETER}$

H(X)	FE
0.843850150-01	0.168770030 02
0.177502000 00	0.186233970 02
0.298547820 00	0.242091650 02
0.429446880 00	0.261798100 02
0.553753020 00	0.248612290 02
0.661479980 00	0.215453930 02
0.749028510 00	0.175097050 02
0.817042030 00	0.136027040 02
0.868236210 00	0.102388360 02
0.983621010 00	0.230769590 02

MAX ABS DIFF (DF0-H(X))= 0.717637920-01

ALPHA= 0.715526420 00 U= 0.476504810 01

CHISQ= 0.312478330 02

NG(CHISQ)=10 SMFE= 0.500000000 01

P(EXCEEDING CHISQ)= 0.559621230-04 DF= 7

THE INCO COMPANY INVESTMENT PROBLEM

FISHER-TIPPETT TYPE 2 DISTRIBUTION

DENSITY FUNCTION $G(X) = \text{GAMMA} / B(X/B)^{**}(-\text{GAMMA}-1) \text{ EXP}(-(X/B)^{**}(-\text{GAMMA}))$

$X=C, B=C, \text{GAMMA}=0$

DISTRIBUTION FUNCTION $H(X) = \text{EXP}(-(X/B)^{**}(-\text{GAMMA}))$

$\text{GAMMA}=\text{SLOPE}, B=\text{SCALE PARAMETER}$

H(X)	FE
0.11007046D 00	0.22014093D 02
0.23078551D 00	0.24143010D 02
0.35972162D 00	0.25787221D 02
0.47684340D 00	0.23424357D 02
0.57512388D 00	0.19656097D 02
0.65453943D 00	0.15883109D 02
0.71768604D 00	0.12629321D 02
0.76766892D 00	0.99965777D 01
0.80729955D 00	0.79261259D 01
0.92642358D 00	0.23824806D 02

MAX ABS DIFF (DFD-F(X))= 0.13270045D 00

GAMMA= 0.30610715D 01 B= 0.45327203D 01

CHISQ= 0.64155743D 02

NG(CHISQ)=10 SMFE= 0.50000000D 01

P(EXCEEDING CHISQ)= 0.22228275D-10 CF= 7

THE INCC COMPANY INVESTMENT PROBLEM

WEIBULL DISTRIBUTION

DENSITY FUNCTION $G(X) = (GAMMA/THETA) X^{**}(GAMMA-1) \exp(-X^{**}GAMMA/THETA)$

X GREATER THAN OR=0, GAMMA GREATER THAN 0, THETA GREATER THAN 0

DISTRIBUTION FUNCTION $H(X) = 1 - \exp(-X^{**}GAMMA/THETA)$

GAMMA=SHAPE PARAMETER, THETA=SCALE PARAMETER

H(X)	FE
0.10972223D 00	0.21944445D 02
0.17791933D 00	0.13639421D 02
0.26694470D 00	0.17805074D 02
0.37428891D 00	0.21468843D 02
0.49370919D 00	0.23884056D 02
0.61577443D 00	0.24413047D 02
0.72966417D 00	0.22777949D 02
0.82584884D 00	0.19236934D 02
0.89864740D 00	0.14559711D 02
0.99980322D 00	0.20231165D 02

MAX ABS DIFF (DFD-H(X))= 0.54722227D-01

GAMMA= 0.39106141D 01

THETA= 0.11543904D 04

CHISQ= 0.27419107D 02

NG(CHISQ)=10

SMFE= 0.50000000D 01

P(EXCEEDING CHISQ)= 0.28008302D-03

CF= 7

THE INCO COMPANY INVESTMENT PROBLEM
SAMPLE STATISTICS BETA DISTRIBUTION

XPB	XPI	XPT	FI	CFI	DFD
C.0	C.111111D CC	0.222222D 00	0.110000C 02	0.110000D 02	0.550000D-01
0.222222D 00	0.250000D 00	0.277778D 0C	0.220000C 02	0.330000D 02	0.165000D 00
C.277778D 0C	C.305556D 00	0.333333C 00	0.150000C 02	0.480000C 02	0.240000D 00
0.333333D 0C	C.361111D 00	0.388889D 00	0.270000D 02	0.750000D 02	0.375000D 00
C.388889D 00	0.416667D 00	0.444444D 0C	0.340000D 02	0.109000D 03	0.545000D 00
C.444444D 0C	0.472222D 0C	0.500000C 00	0.230000D 02	0.132000D 03	0.660000D 00
0.500000D 00	0.527778D 00	C.555556D 00	0.200000C 02	0.152000D 03	0.760000D 00
C.555556D 00	0.583333D 00	0.611111D 00	0.130000C 02	0.165000D 03	0.825000D 00
0.611111D 0C	C.638889D 00	0.666667C 00	0.230000C 02	0.188000D 03	0.940000D 00
0.666667D 00	0.833333D 00	0.100000D 01	0.120000D 02	0.200000D 03	0.100000D 01

THE INCO COMPANY INVESTMENT PROBLEM

BETA DISTRIBUTION

DENSITY FUNCTION $G(X) = (\text{GAMMA}(\text{LAMBDA} + \text{THETA}) / (\text{GAMMA}(\text{LAMBDA}) * (\text{GAMMA}(\text{THETA})))) * X^{**}(\text{LAMBDA}-1) * (1-X)^{**}(\text{THETA}-1)$

X GREATER OR=0 AND X LESS THAN OR =1, LAMBDA GREATER THAN 0, THETA GREATER THAN 0

DISTRIBUTION FUNCTION $H(X) = (\text{GAMMA}(\text{LAMBDA} + \text{THETA}) / (\text{GAMMA}(\text{LAMBDA}) * (\text{GAMMA}(\text{THETA})))) \text{ INTEGRAL } (X^{**}(\text{LAMBDA}-1) * (1-X)^{**}(\text{THETA}-1) DX)$

LAMBDA, THETA=SHAPE PARAMETERS

H(X)

FE

0.96635409D-01	0.19327082D 02
0.17581262D 00	0.15835443D 02
0.27599004D 00	0.20035484D 02
0.39036675D 00	0.22875341D 02
0.51052548D 00	0.24031746D 02
0.62780049D 00	0.23455002D 02
0.73444163D 00	0.21328228D 02
0.82455934D 00	0.18031543D 02
0.89488955D 00	0.14058041D 02
0.10000000D 01	0.21022091D 02

THETA= 0.43003284D 01

LAMBDA= 0.34421984D 01

M1= 0.44458333D 00

M2= 0.28244579D-01

MAX ABS DIFF (DFO-H(X))= 0.45110453D-01

CHISQ= 0.23186910D 02

NG(CHISQ)=10

SMFE= 0.50000000D 01

P(EXCEEDING CHISQ)= 0.15814618D-02

DF= 7

B.14 The INCO Company Investment Problem--

RUM Input

DATA SHEET

PROBLEM The Inco Company Investment Utility PAGE 1 OF 1 DATE 10/20/73 WRITTEN 82

01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80																																																																																
CASE IDENTIFICATION TITLE																																																																																																																																																															
THE INCO COMPANY INVESTMENT PROBLEM																																																																																																																																																															
REQUESTED UTILITY MODEL 1 = REQUESTED 0 = NOT REQUESTED																																																																																																																																																															
FARRER																				CRAMER AND SMITH																				BUSSEY																																																																																																																							
0																				0																				1																																																																																																																							
FARRER: $U = 1 - \exp(-A \cdot V)$																																																																																																																																																															
A																				V																																																																																																																																											
CRAMER AND SMITH: $U = 1 - A \cdot (S \cdot B) \cdot (P \cdot C)$																																																																																																																																																															
A																				B																				C																				V																				S																				P																																																											
BUSSEY: $U = A \cdot V_1 - B \cdot V_2 - C \cdot V_3 - D \cdot V_4$																																																																																																																																																															
A																				B																				C																				D																				V1																				V2																				V3																				V4																			
48.0																				8.0																				2.75																				0.12																				0.50125																				2.5391																				4.63678																				21.6394																			

B.15 The INCO Company Investment Problem --

RUM Output

```
*****
*
*
*
*      THE INCO COMPANY  INVESTMENT PROBLEM
*
*
*
*****
```

B U S S E Y S MODEL: UTILITY= 0.139E 02

VITA

Mohamed Sadek Eid

Candidate for the Degree of

Doctor of Philosophy

Thesis: A UTILITY PROGRAM FOR THE ANALYSIS OF RISK (UPFAR)

Major Field: Industrial Engineering and Management

Biographical:

Personal Data: Born in Cairo, Egypt, October 20, 1938, the son of Mr. and Mrs. Sadek M. Eid.

Education: Graduated from Khediwiah Secondary School, Cairo, Egypt, in 1955; received Bachelor of Science degree in Production Engineering from Cairo University in 1960; received Diploma in Planning from the Institute of National Planning, Cairo, Egypt, in 1962; received Master of Engineering degree in Industrial Engineering from Dartmouth College, Hanover, New Hampshire, in 1968; received Industrial Engineer degree from Columbia University, New York, New York, in 1973; completed requirements for the Doctor of Philosophy degree at Oklahoma State University in July, 1974.

Professional Experience: Operations Research Analyst, The Institute of National Planning, Cairo, Egypt (1960-1964); Mechanical Engineer, Mobiloil Company, Cairo, Egypt (1964-1965); Operations Research Analyst, The Southern New England Telephone Company, New Haven, Connecticut (1966-1967); Operations Research Analyst, Mobiloil Company, New York, New York (Summer 1967); Operations Research Analyst, Consultant, Officer and Assistant Treasurer, Chemical Bank, New York, New York (1968-1974).